National Science Center
Kharkov Institute of Physics and Technology

Bent crystal as an efficient tool for high-energy particle beam control

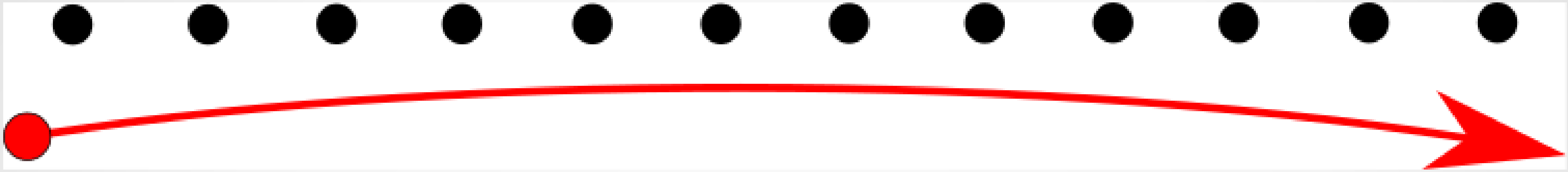
I.V. Kyryllin

Akhiezer Institute for Theoretical Physics of National Science Center "Kharkov Institute of Physics and Technology", Kharkiv, Ukraine

Karazin Kharkiv National University, Kharkiv, Ukraine

TES HEP, Poltava, 2018

Charged-particle motion in the field of crystal atomic strings



$$N_c Z Z_p e^2/\hbar c \gg 1$$

$$N_c = R/(a\psi)$$

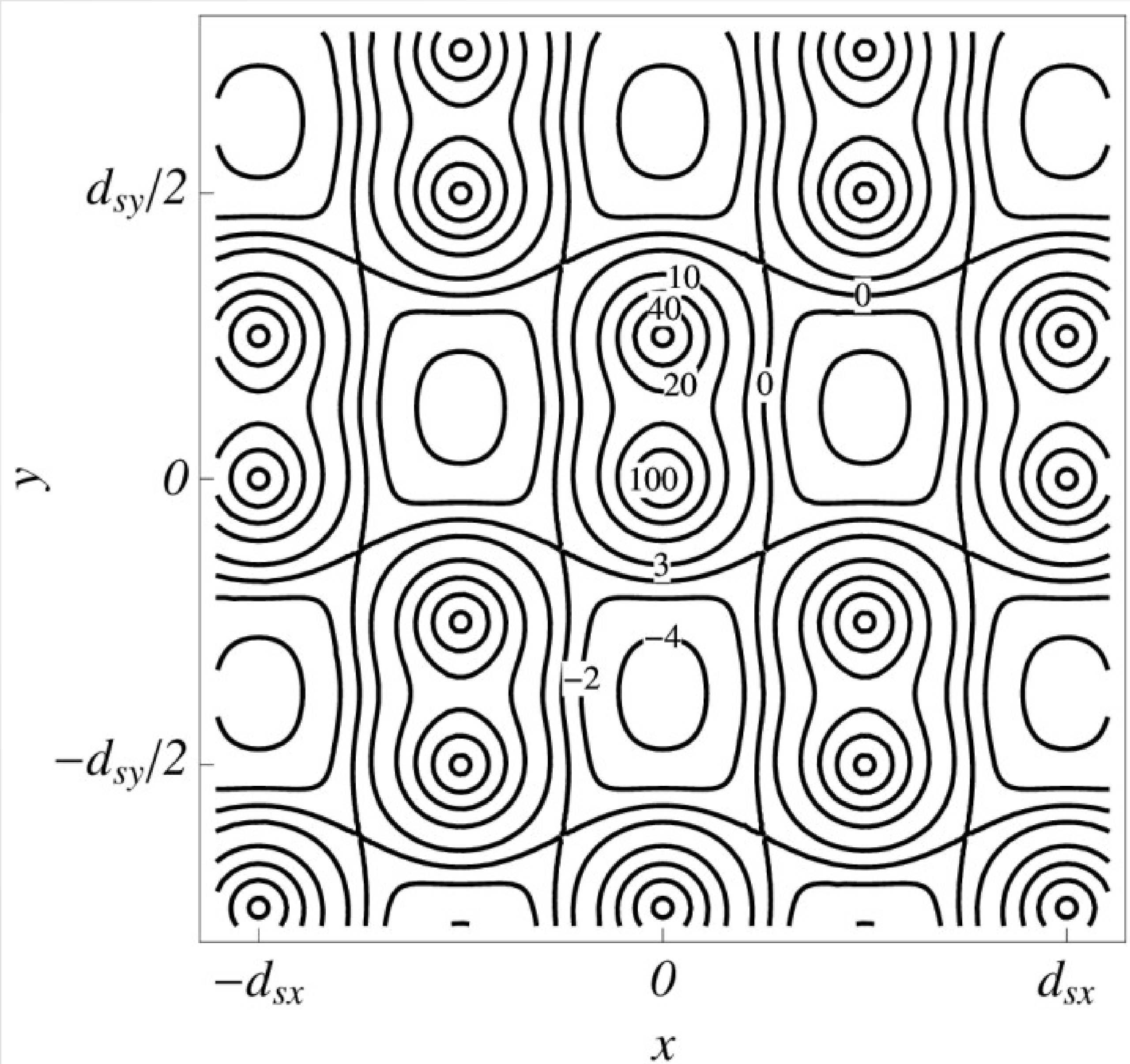
$$\frac{d}{dt}\frac{m\boldsymbol{v}}{\sqrt{1-v^2/c^2}} = -q\nabla\Phi_c(\boldsymbol{r})$$

$$\Phi_c(\boldsymbol{r}) = \sum_n \Phi_a(\boldsymbol{r}-\boldsymbol{r}_n)$$

$$\Phi(\boldsymbol{\rho}) = \frac{1}{L}\int_{-\infty}^{\infty} dz\,\Phi_c(\boldsymbol{\rho}, z)$$

$$\ddot{\boldsymbol{\rho}} = -\frac{c^2 q}{E_{\parallel}}\frac{\partial}{\partial\boldsymbol{\rho}}\Phi(\boldsymbol{\rho})$$

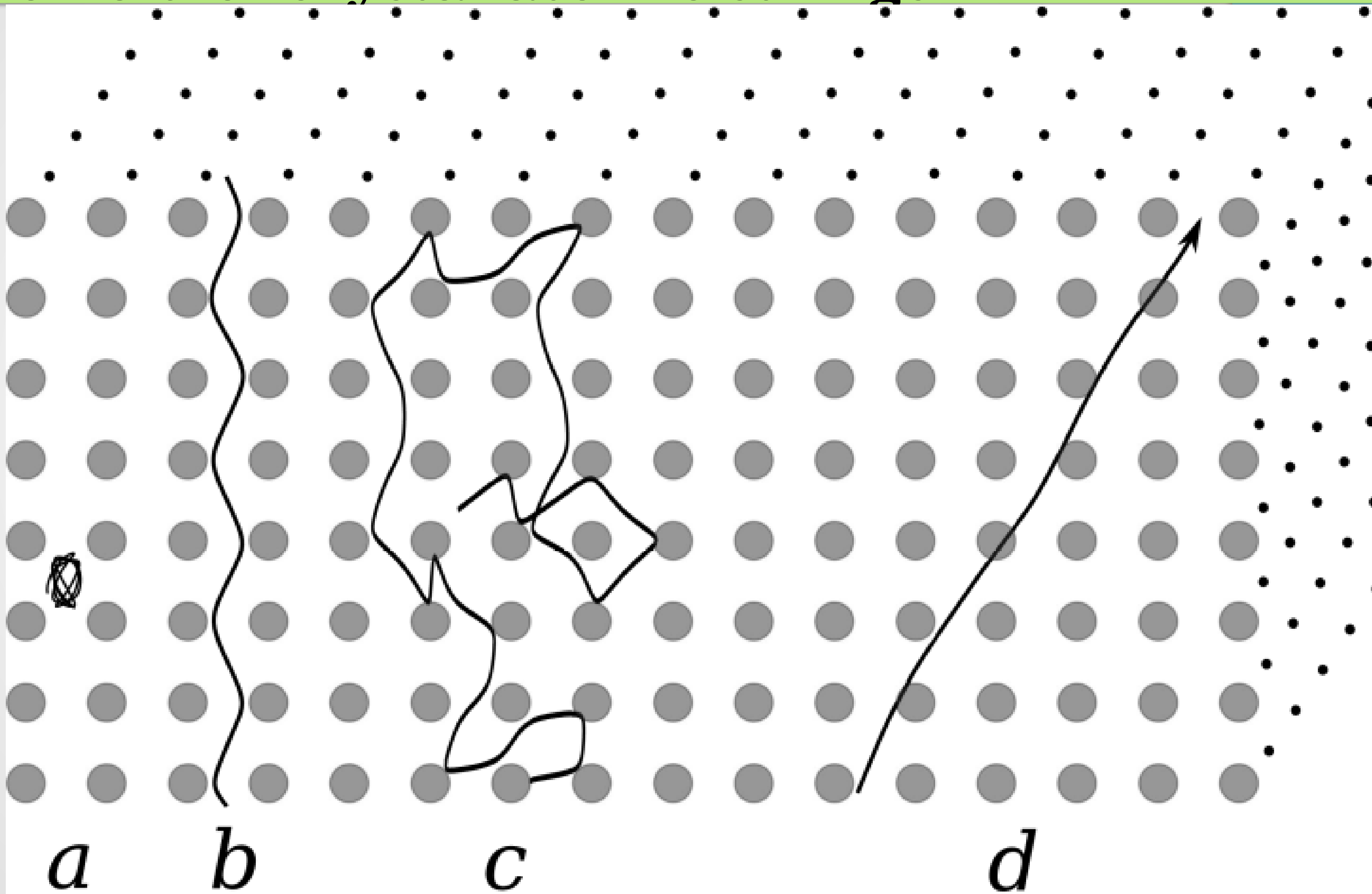
Charged-particle motion in the field of crystal atomic strings



$$\Phi(\boldsymbol{\rho}) = \frac{1}{L}\int_{-\infty}^{\infty} dz\, \Phi_c(\boldsymbol{\rho}, z)$$

$$\varepsilon_{\perp} = \frac{E\dot{\boldsymbol{\rho}}^2}{2c^2} + U(x, y)$$

Regimes of charged-particle motion in the field of crystal atomic strings



a) axial channeling

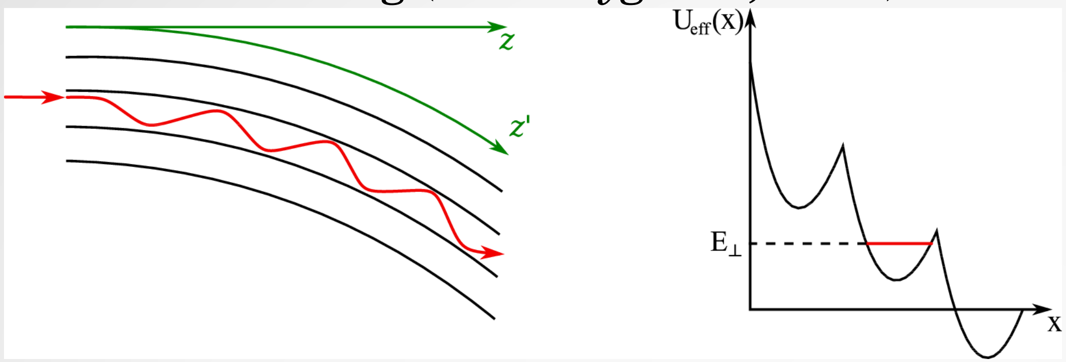
b) planar channeling

c) stochastic scattering

d) strongly above-barrier motion

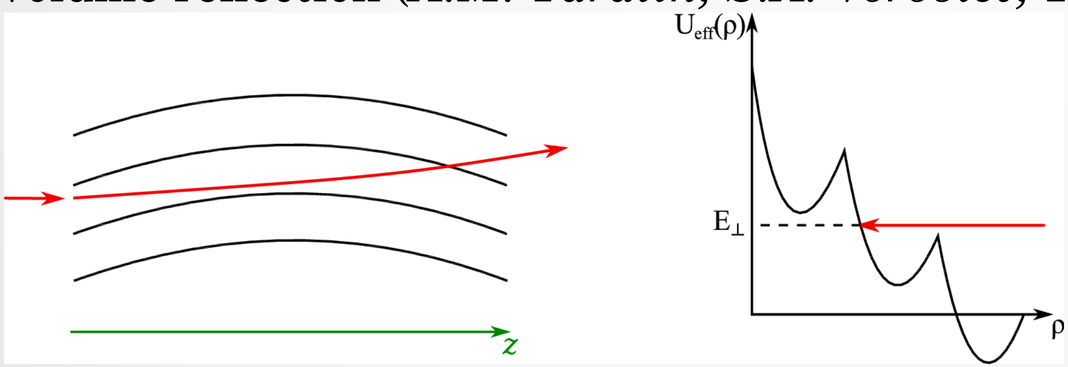
Mechanisms of deflection of charged particles by a bent crystal

- Planar channeling (*E.N. Tsyganov, 1976*)



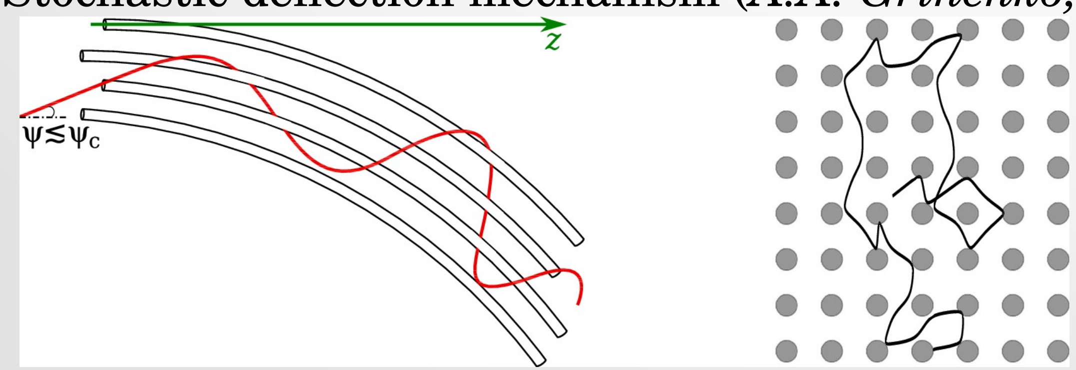
1979 — JINR
1980 — CERN

- Volume reflection (*A.M. Taratin, S.A. Vorobiev, 1987*)



2006 — IHEP
2006 — PNPI
2007 — CERN

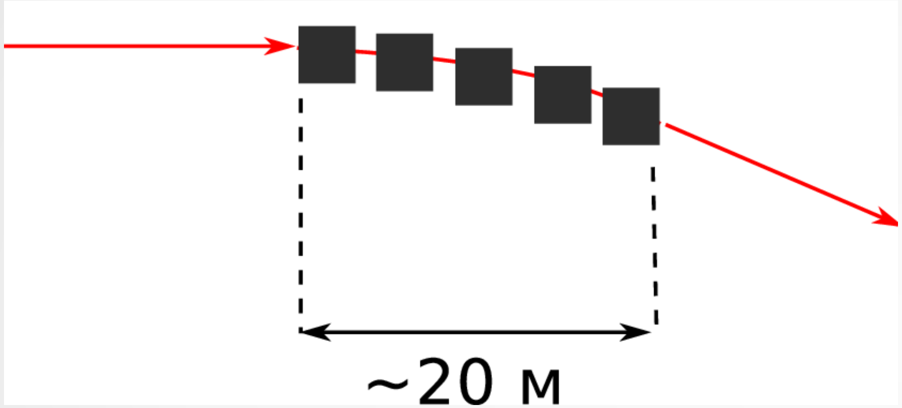
- Stochastic deflection mechanism (*A.A. Grinenko, N.F. Shul'ga, 1991*)



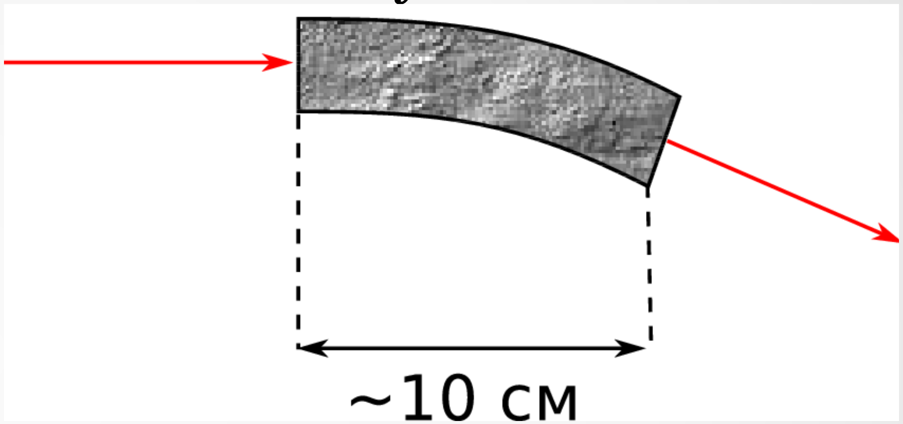
2008 — CERN, protons
2009 — CERN, π^--mesons

Charged particle beam deflection

Magnets

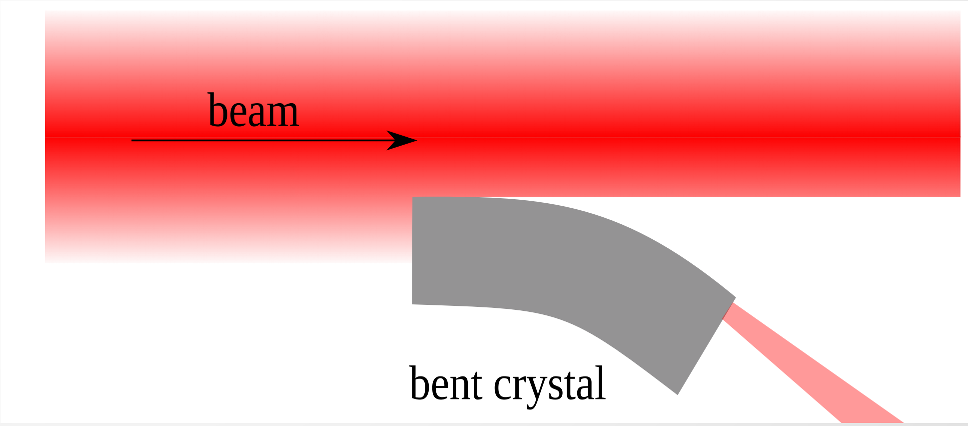


Bent crystal



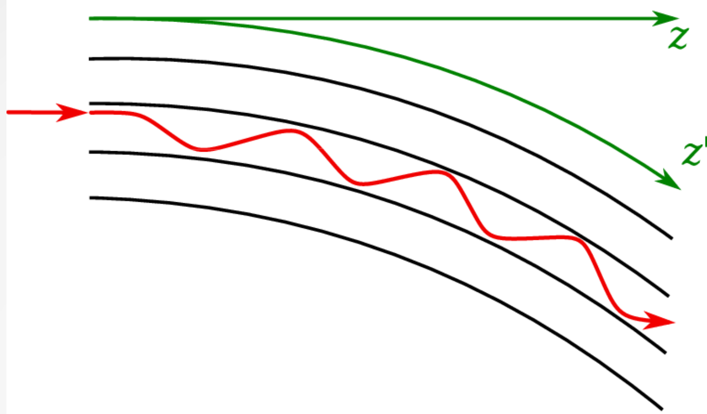
Advantages of bent crystals in front of magnets:

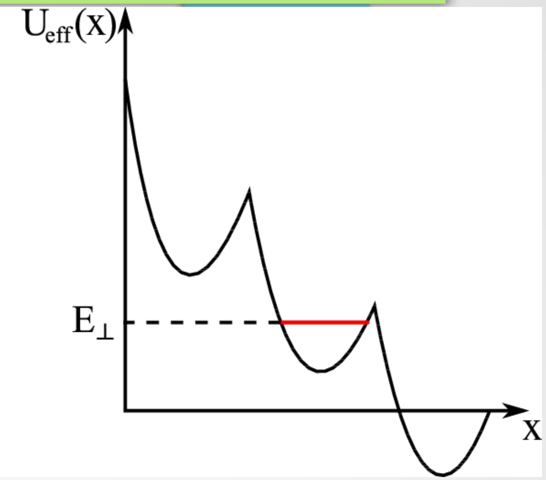
- compact size
- do not need electric power consumption
- do not need cooling

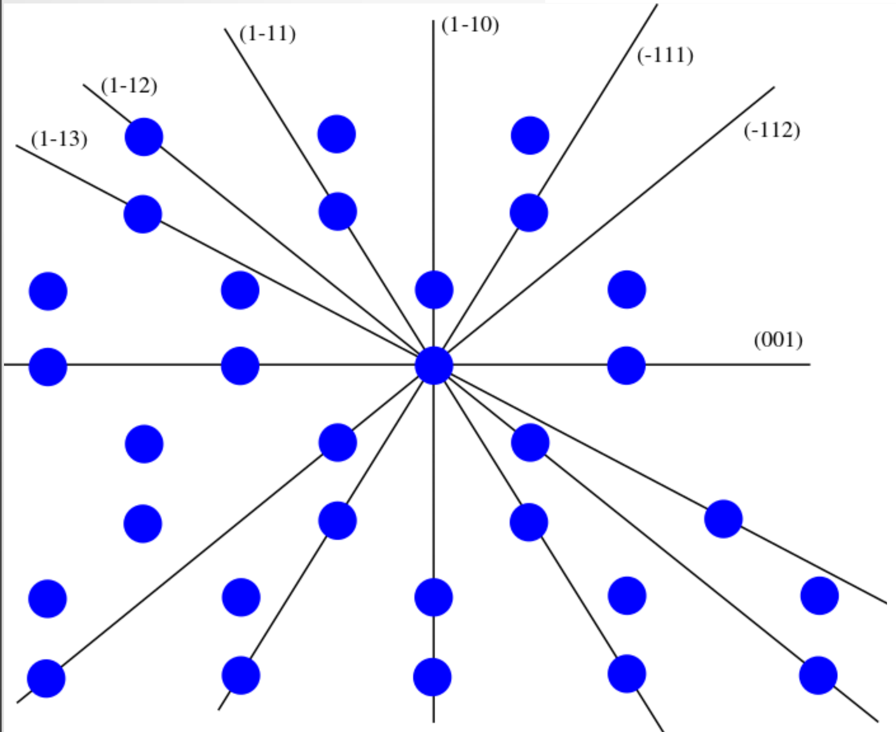


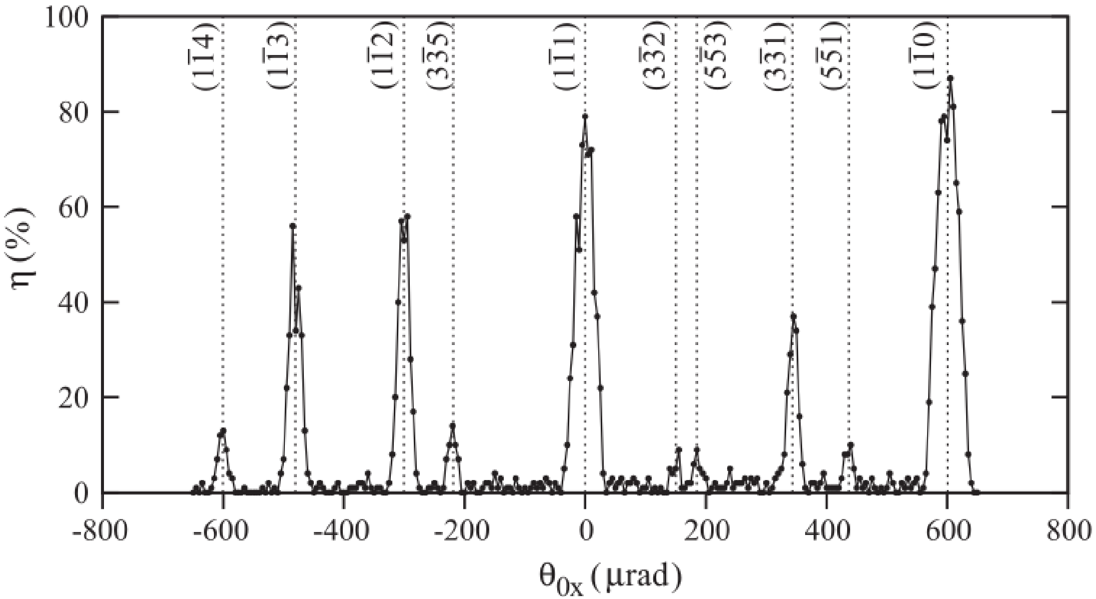
The electric field strength ~10^{10} V/cm

Planar channeling in a bent crystal

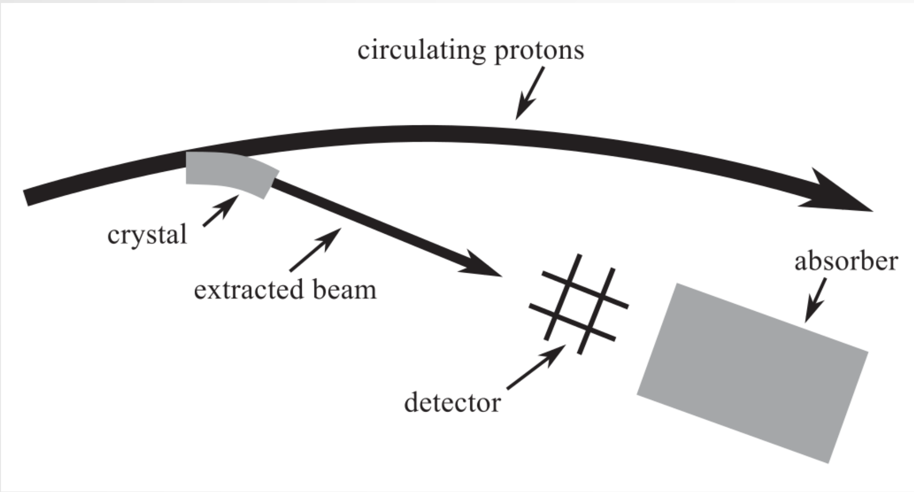


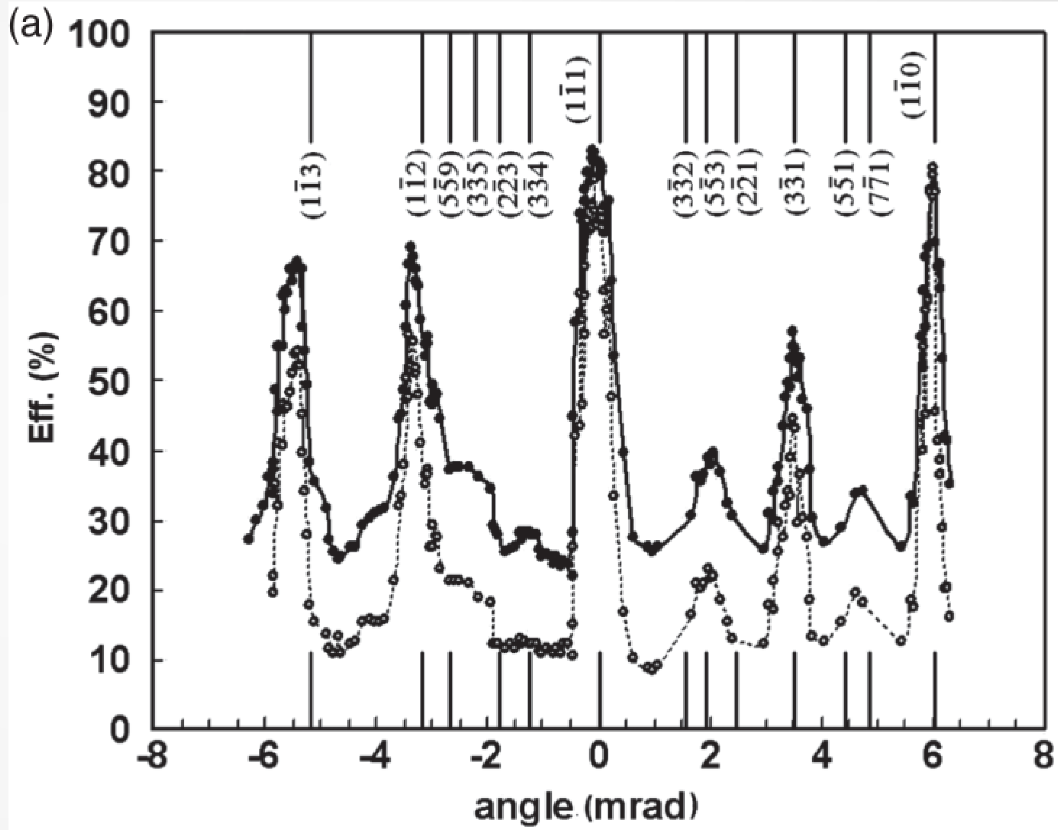






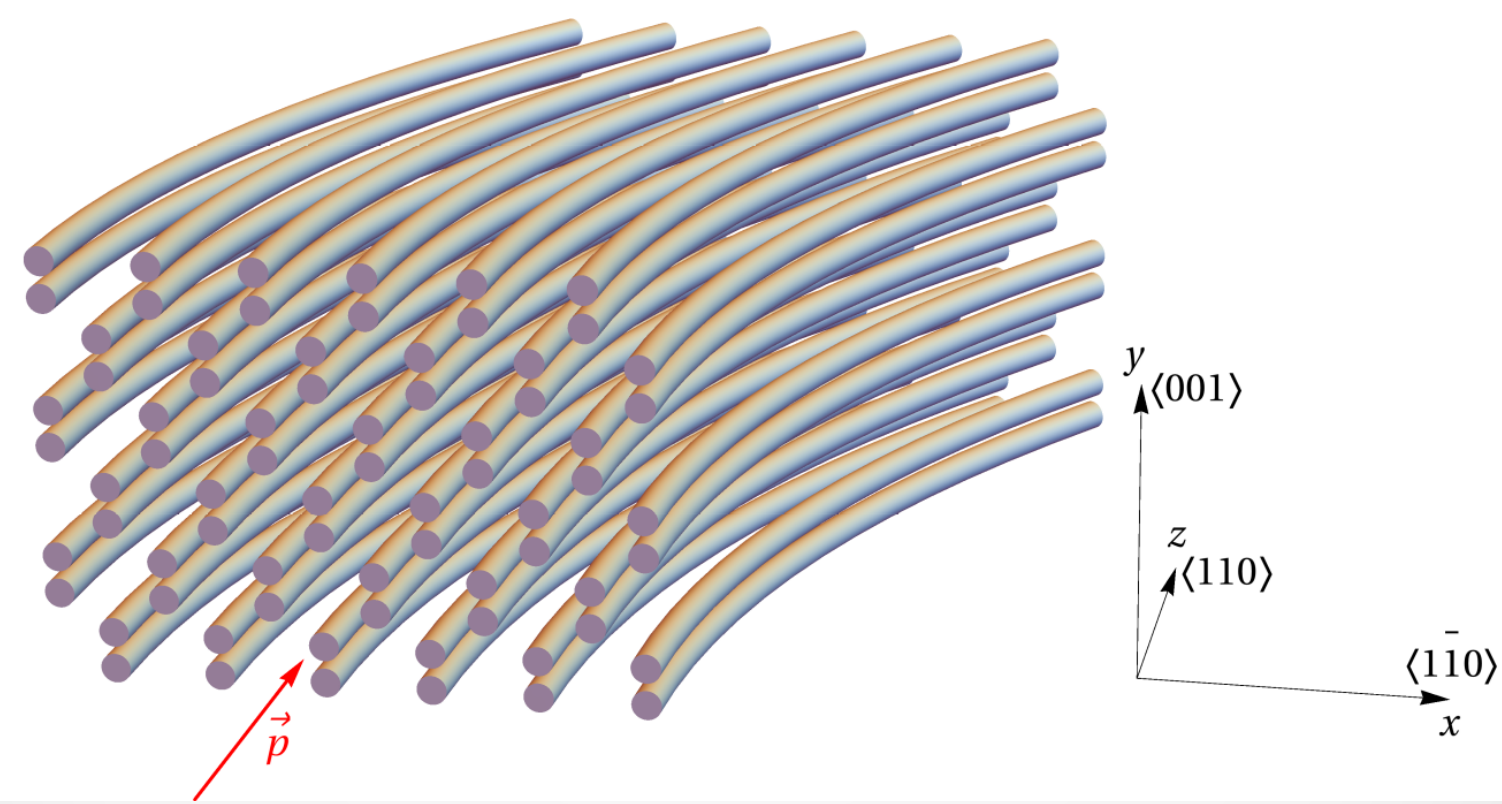
Planar channeling in a bent crystal





A.G. Afonin et al. Phys. Rev. ST Accel. Beams 15 (2012) 081001

Stochastic deflection mechanism



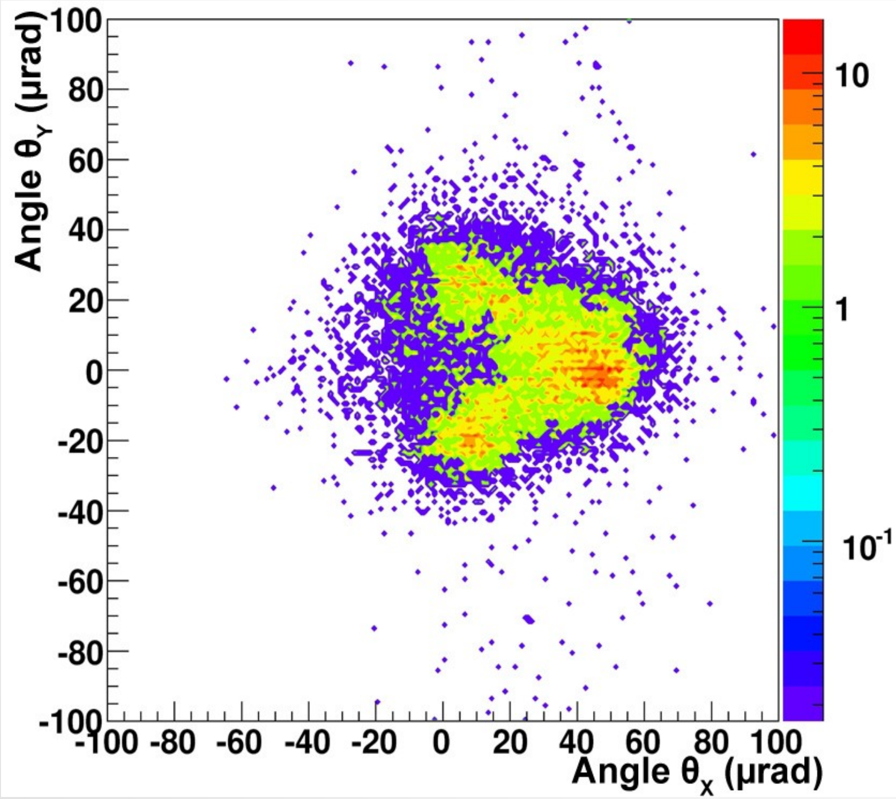
Greenenko-Shul'ga criterion: $$\frac{l}{R\psi_c}\frac{L}{R\psi_c}<1$$

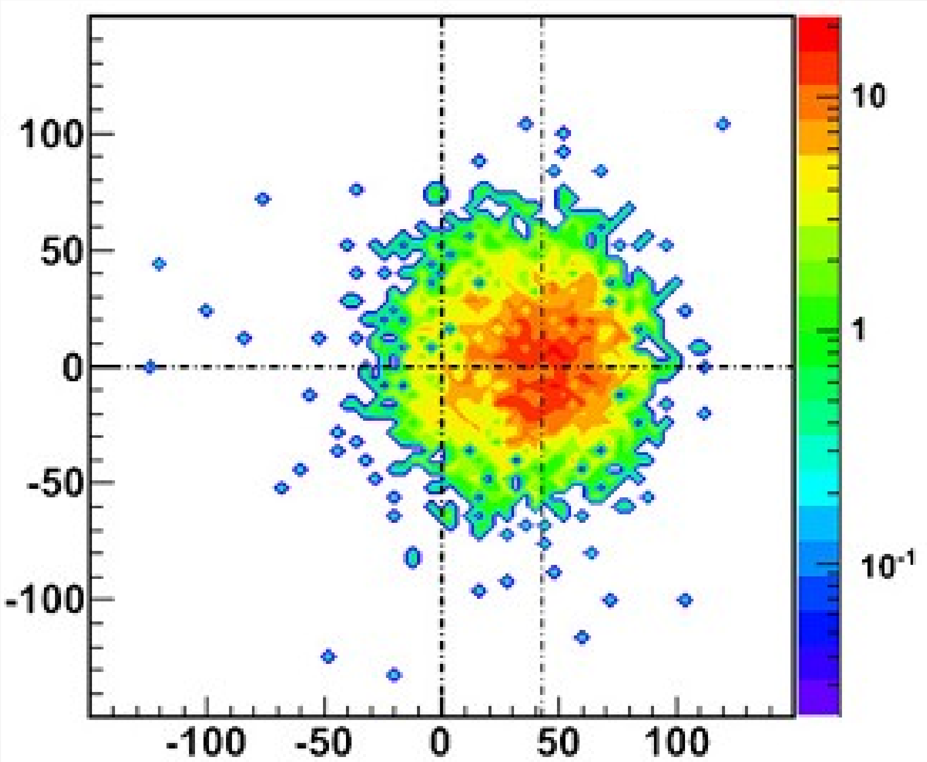
R is crystal curvature radius;
ψ_c is the critical angle of axial channeling;
l is the mean free path between successive collisions with atomic strings;
L is the thickness of the crystal.

Stochastic mechanism of high energy charged particles deflection by bent Si crystal with R=40 m

CERN experiment, UA9 collaboration

p^+, E=400 GeV **π^-, E=150 GeV**

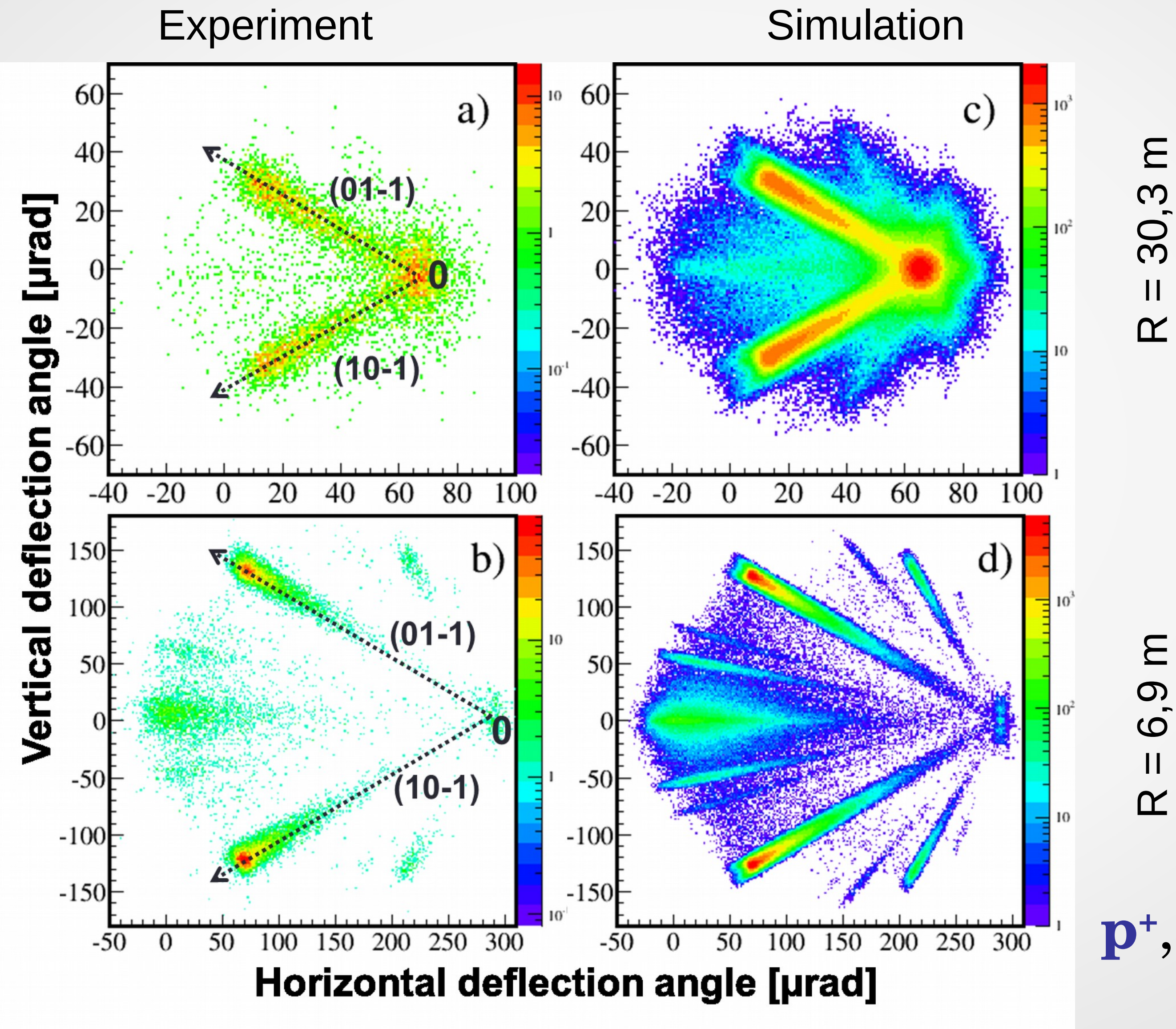




W. Scandale et al. Phys. Rev. Lett. 101 (2008), 164801

W. Scandale et al. Physics Letters B 680 (2009) 301

Stochastic deflection and beam splitting

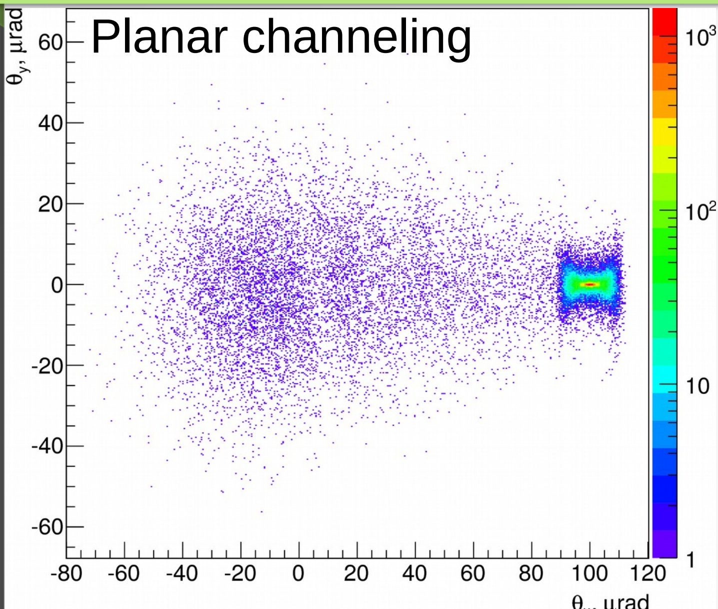


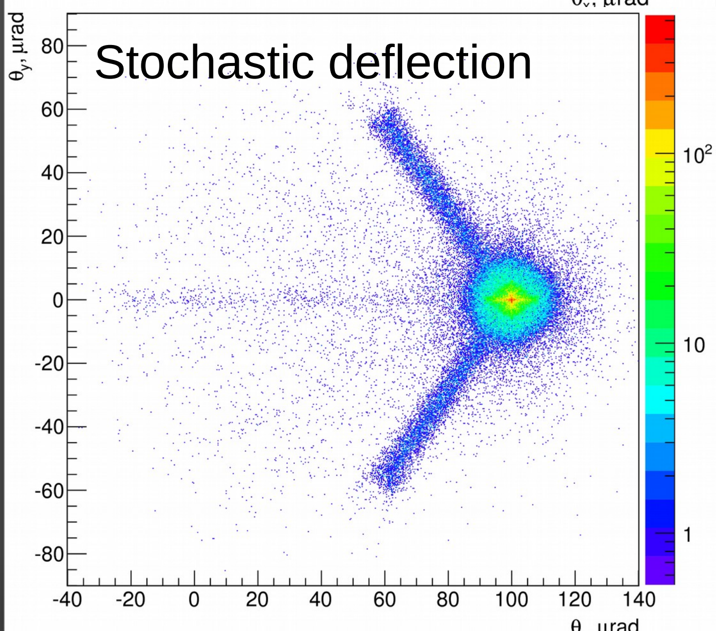
p^+, E=400 GeV

V. Guidi, I.V. Kirillin, N.F. Shul'ga et al. Eur. Phys. J. C (2016) 76:80

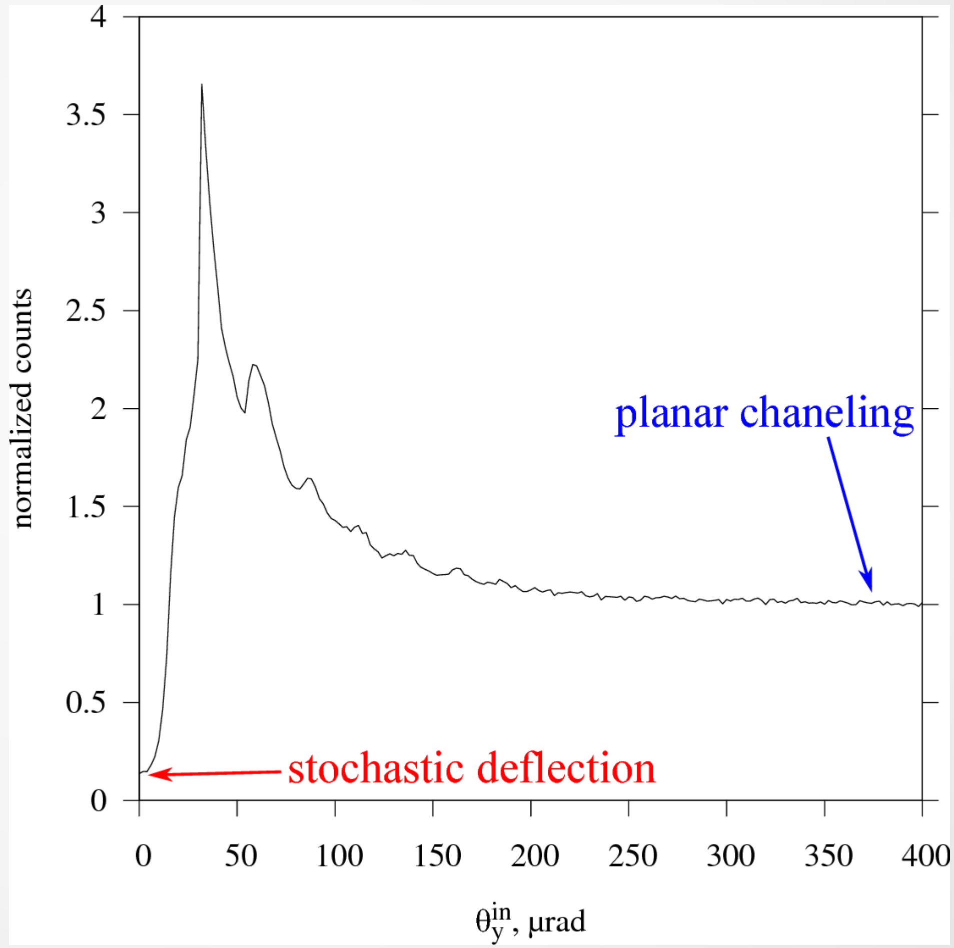
Close collisions probability

$$\frac{W_{pl}}{W_{ax}} = \frac{a}{\pi r_T} \gg 1$$





p^+, 270 GeV, Si <110>, L = 5 mm, R = 50 m



Yu.A. Chesnokov, I.V. Kirillin, W. Scandale, N.F. Shul'ga, V.I. Truten' . Physics Letters B 731 (2014) 118

Experimental verivication

(W. Scandale et al. / Physics Letters B 760 (2016) 826)

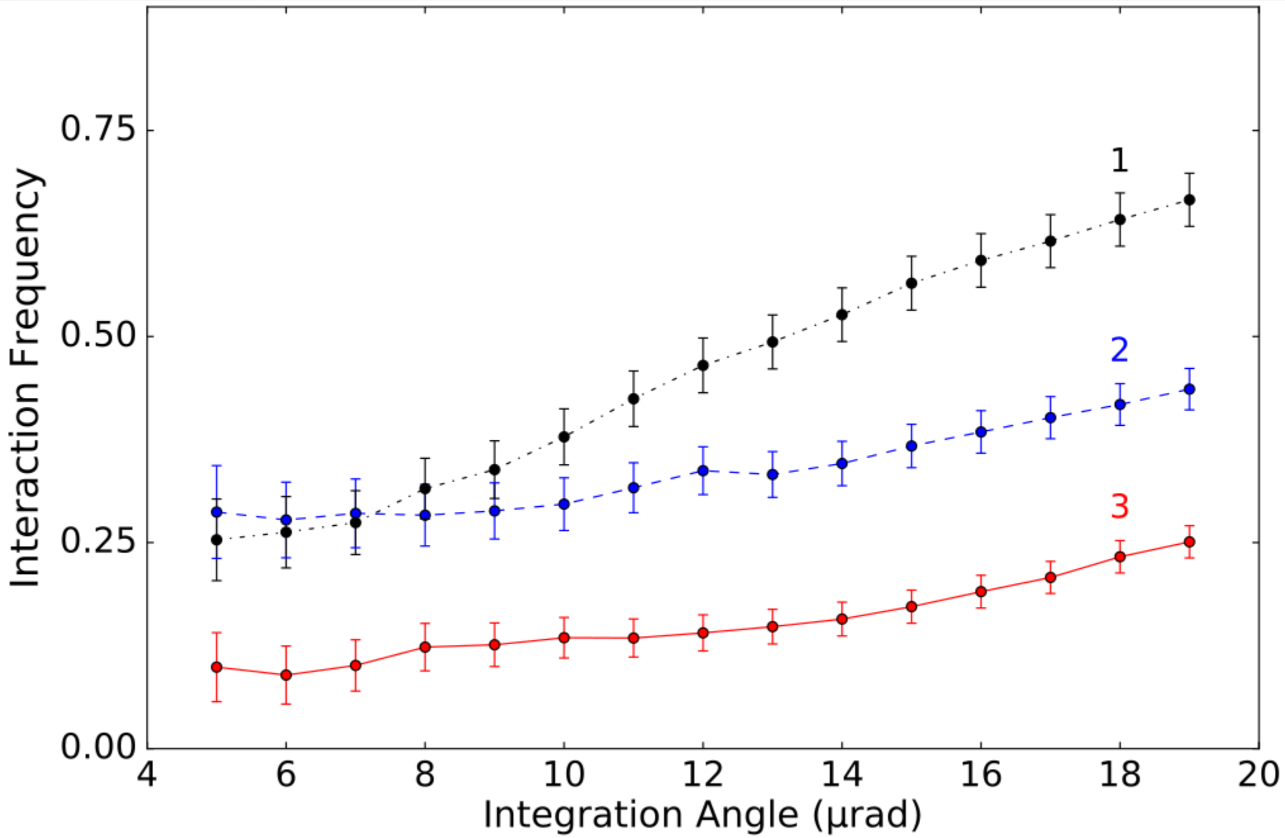


Fig. 5. Measured inelastic nuclear interaction (INI) frequency of 400 GeV/c protons interacting with the ⟨111⟩ and ⟨110⟩ crystals as a function of the angular region around the (110) planar channeling (black dash-dotted line, 1), the ⟨111⟩ axial channeling (blue dashed line, 2) and ⟨110⟩ (red continuous line, 3) orientations. The values are normalized to the INI frequencies for the amorphous crystal orientation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Influence of incoherent scattering on stochastic deflection of high-energy negative particle beams in bent crystals

$$\frac{d}{dz}\overline{\psi^2} = \frac{l}{R^2} + \frac{d}{dz}\overline{\psi_{inc}^2}$$

For $U_{str}(\rho) = U_0\left(\frac{a}{\rho}\right)^2$ $\quad l \approx \frac{1}{4nda}\sqrt{\frac{E}{U_0}}$ $\quad\Rightarrow\quad$ $\overline{\psi^2} = \frac{lL}{R^2} + \overline{\psi_{inc}^2}$

$$\overline{\psi_{inc}^2} = \xi L$$

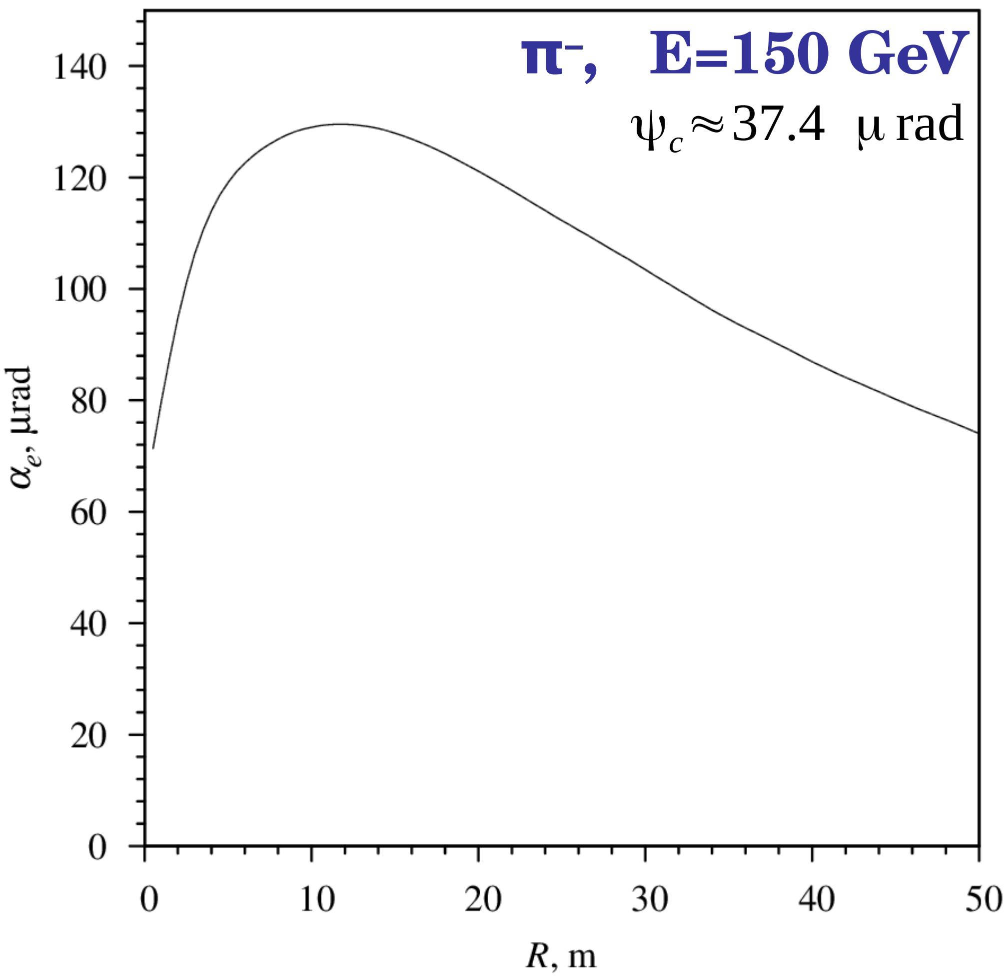
$$L_{st} = \frac{\psi_m^2}{l/R^2 + \xi}$$

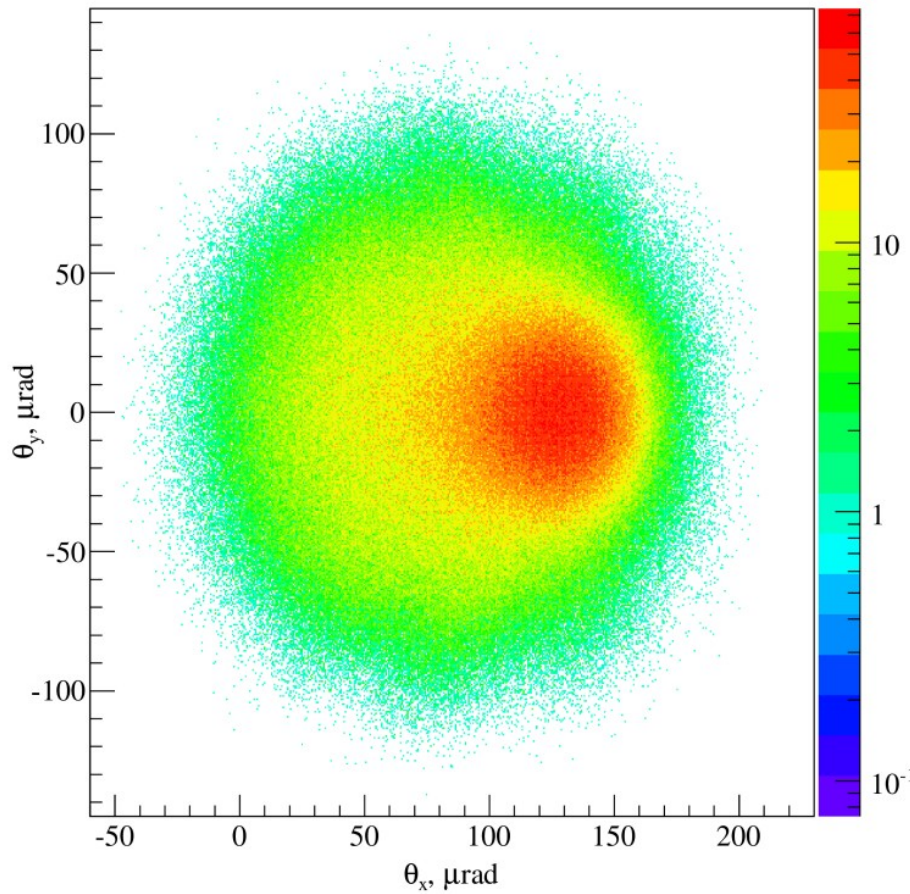
$$\alpha_{st} = \frac{L_{st}}{R} = \frac{\psi_m^2}{l/R + \xi R}$$

For $U_{str}(\rho) = U_0\left(\frac{a}{\rho}\right)^2$ $\quad R_{opt} = \sqrt{\frac{l}{\xi}}$

I.V. Kirillin, N.F. Shul'ga, L. Bandiera, V. Guidi, A. Mazzolari
Eur. Phys. J. C 77 (2017) 117

Influence of incoherent scattering on stochastic deflection of high-energy negative particle beams in bent crystals





π^-, E=150 GeV, Si <110>, L=1.52 mm, R=11.7 m

I.V. Kirillin, N.F. Shul'ga, L. Bandiera, V. Guidi, A. Mazzolari
Eur. Phys. J. C 77 (2017) 117

Dependence of the efficiency of stochastic mechanism of charged particle beam deflection on the particle energy

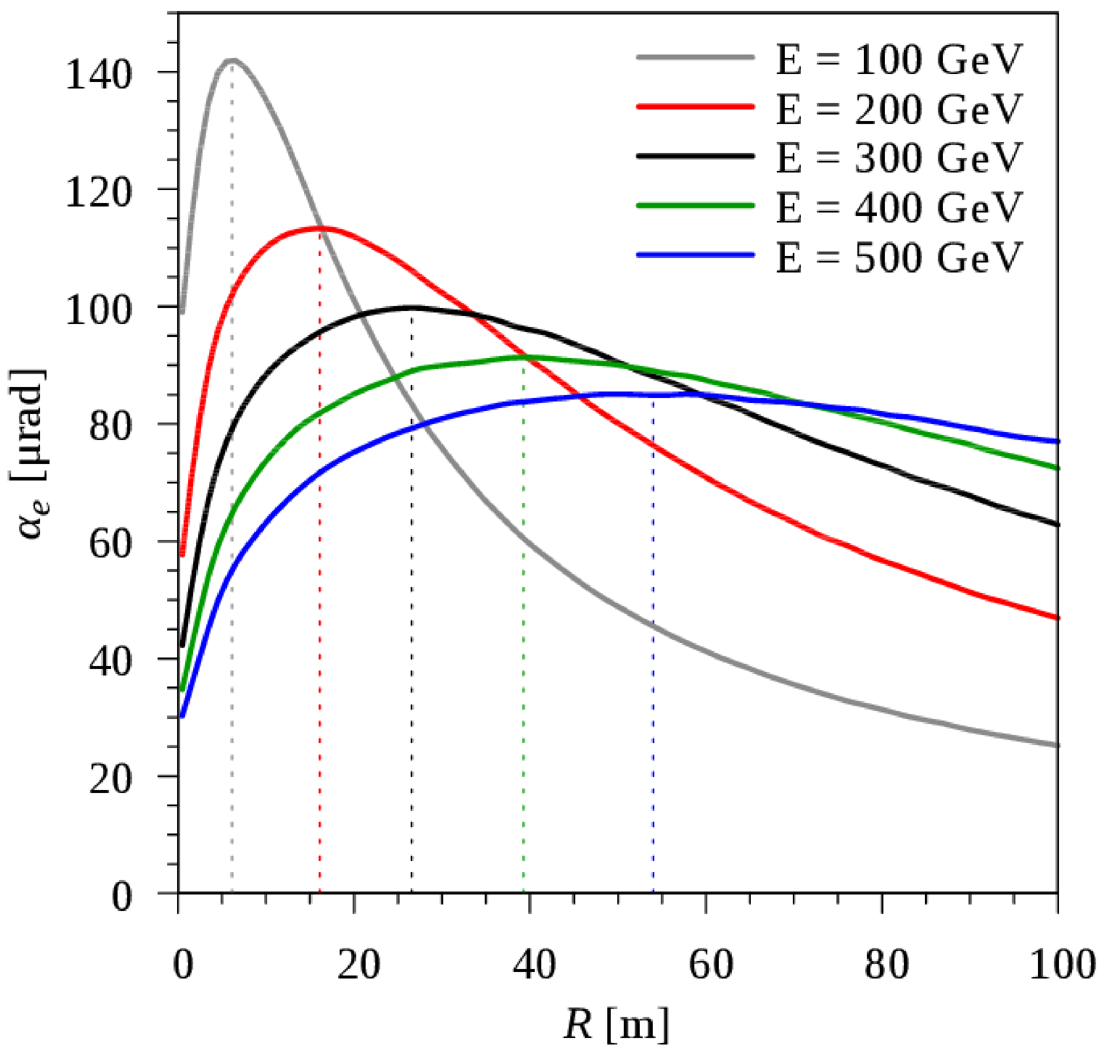
$$\alpha_{st} = \frac{\psi_m^2}{l/R + \xi R}$$

$$\psi_m^2 \propto \frac{1}{E}, \qquad \xi \propto \frac{1}{E^2}$$

For $U_{str}(\rho) = U_0 \left(\frac{a}{\rho}\right)^2 \qquad l \propto \sqrt{E}$

$$R_{opt} \propto \sqrt{\frac{l}{\xi}} \propto E^{5/4}$$

$$\alpha_e \propto \frac{E^{-1}}{E^{-3/4}} \propto E^{-1/4}$$

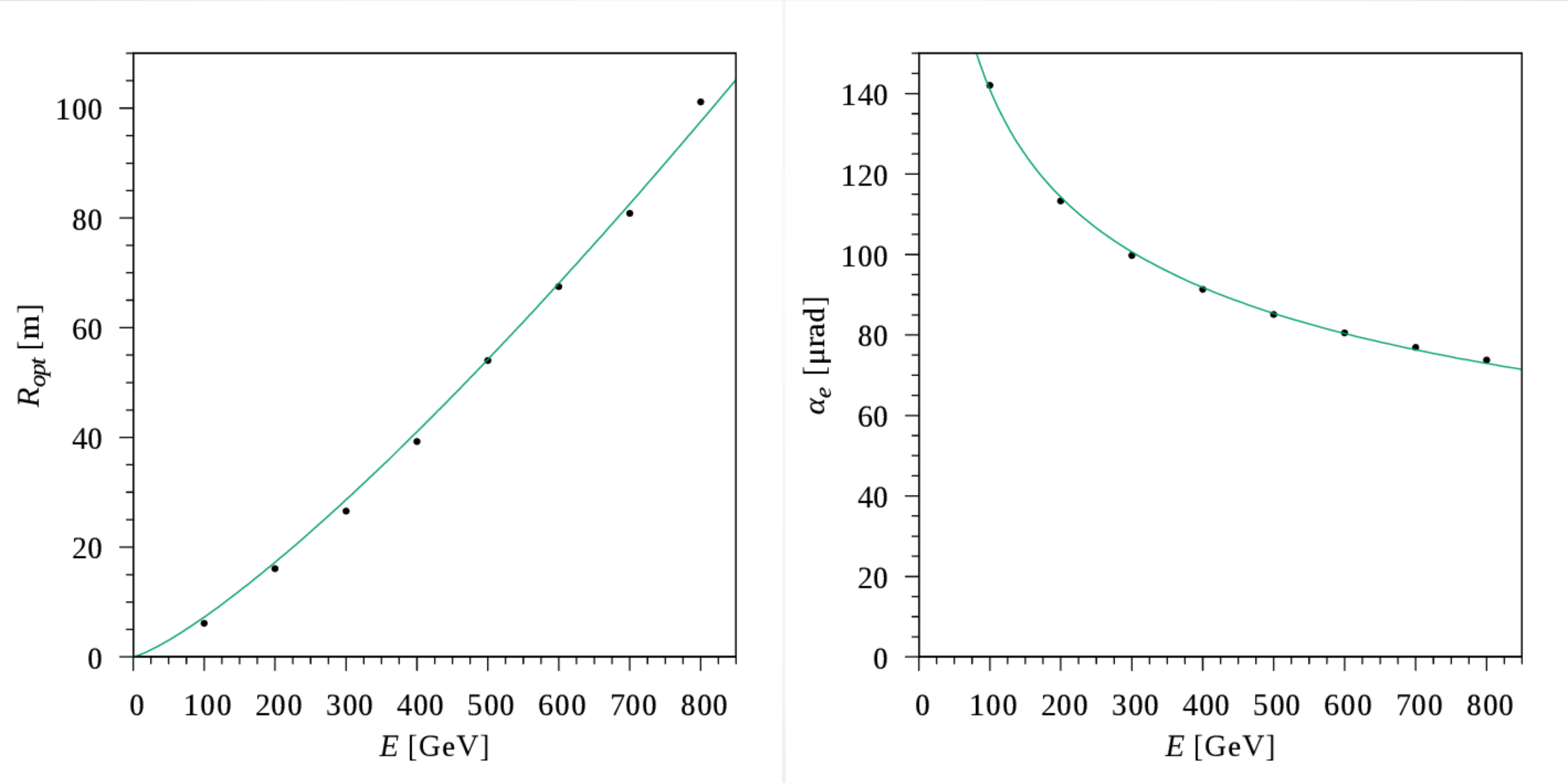


I.V. Kirillin, Probl. Atom. Sci. Tech. 68 (2017) 67

Dependence of the efficiency of stochastic mechanism of charged particle beam deflection on the particle energy

$$R_{opt} \propto \sqrt{\frac{l}{\xi}} \propto E^{5/4}$$

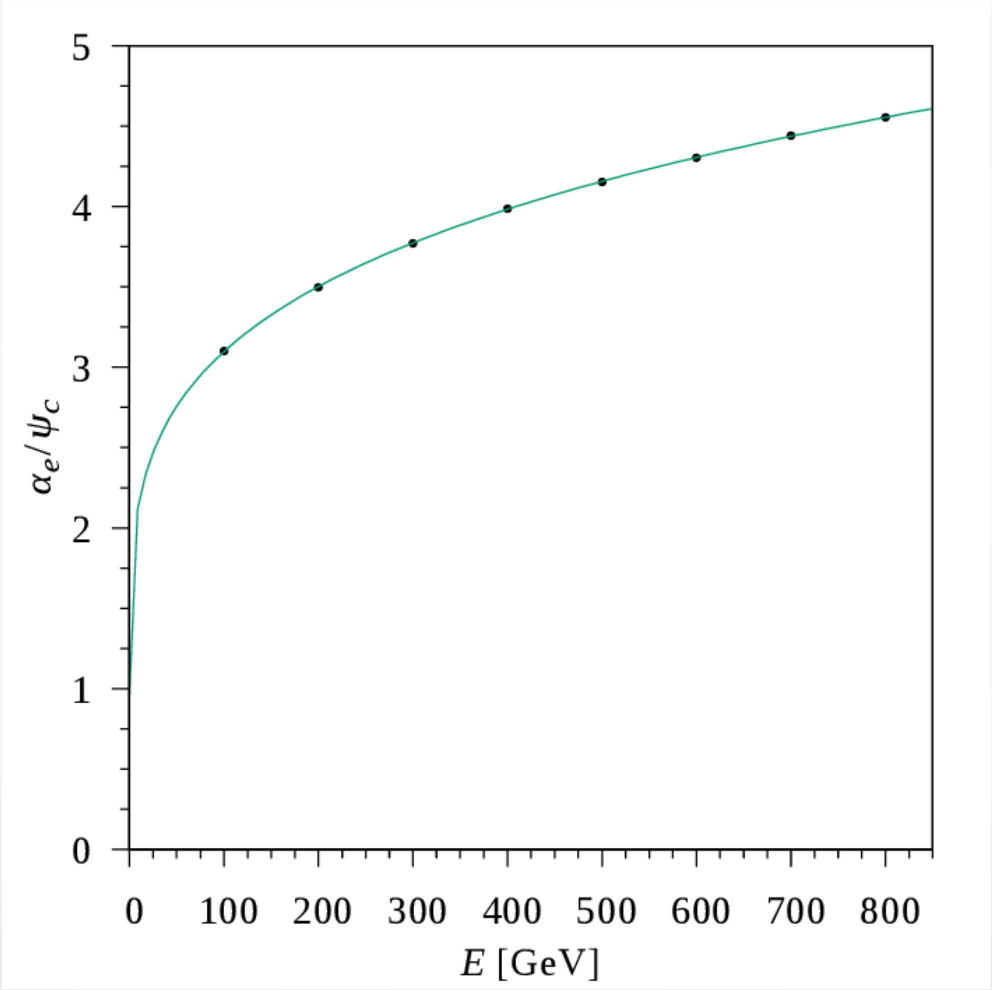
$$\alpha_e \propto E^{-1/4}$$



I.V. Kirillin, Probl. Atom. Sci. Tech. 68 (2017) 67

Dependence of the efficiency of stochastic mechanism of charged particle beam deflection on the particle energy

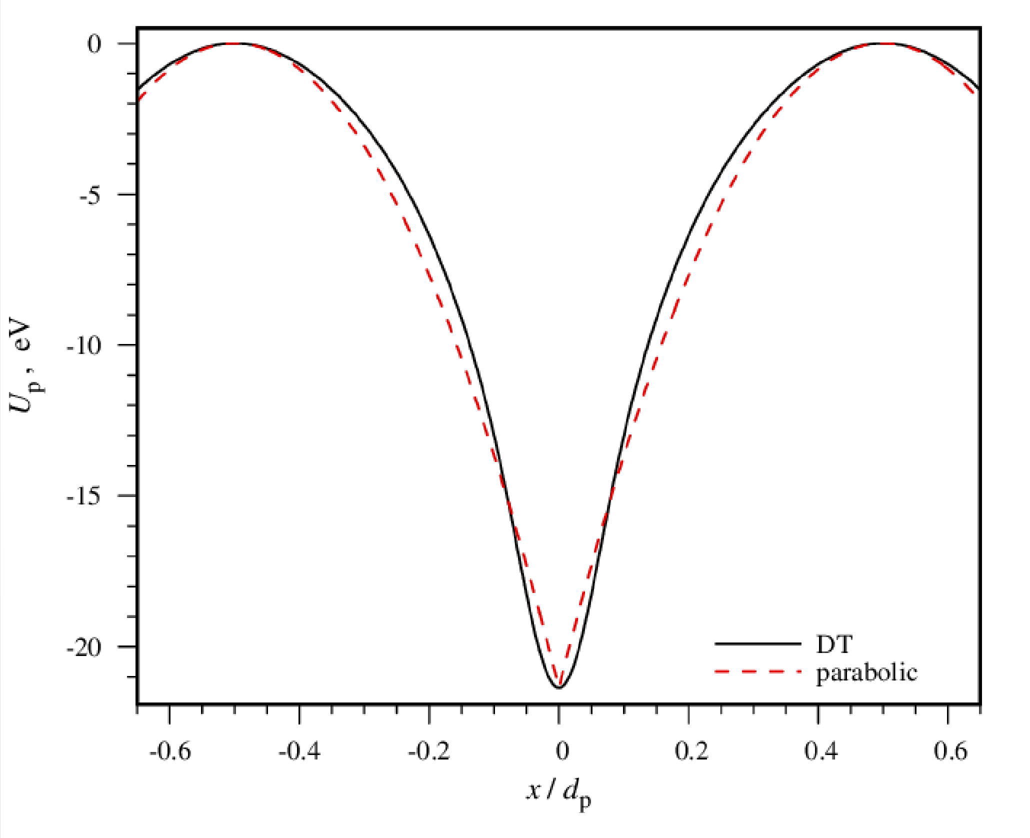
$$\alpha_e/\psi_c \propto E^{1/4}$$



I.V. Kirillin, Probl. Atom. Sci. Tech. 68 (2017) 67

Influence of incoherent scattering on planar channeling of high-energy negative particle beams in bent crystals

$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{d}{dx}\left(U_p(x) + E\frac{x}{R}\right)$$



$$f = \left(1 - \sqrt{\frac{R}{R_c}}\right)\frac{1}{\sqrt{2\pi}\,\theta_0}\int_{-\theta_c}^{\theta_c} \exp\left(-\frac{\theta^2}{2\theta_0^2}\right) d\theta = \\ = \left(1 - \sqrt{\frac{R}{R_c}}\right)\mathrm{erf}\left(-\frac{\theta_c}{\sqrt{2}\,\theta_0}\right)$$

if $\theta_0 = \xi\sqrt{L}$, then

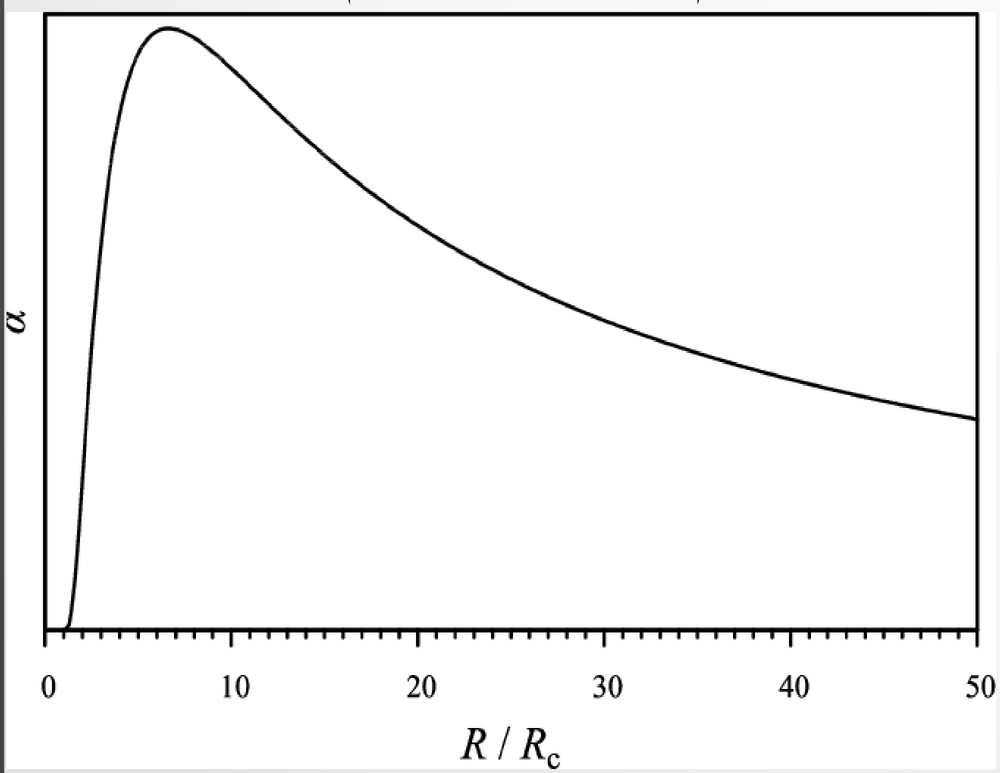
$$L = \frac{\theta_c^2}{2\xi^2\left(\mathrm{erf}^{-1}\left(\frac{f}{1-\sqrt{\frac{R_c}{R}}}\right)\right)^2}$$

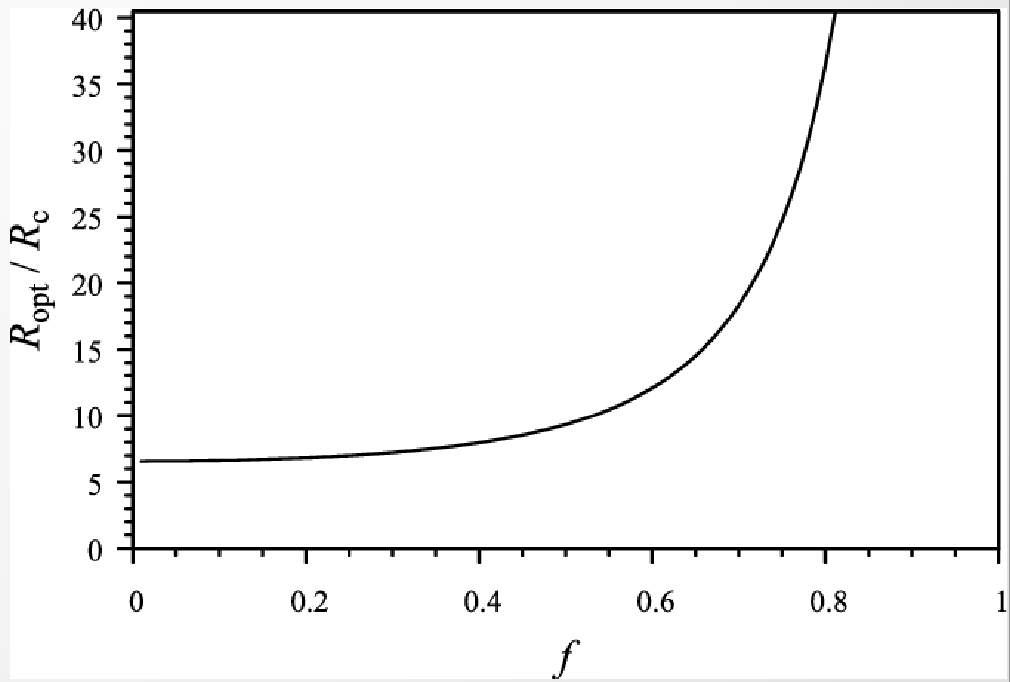
I.V. Kirillin, Phys. Rev. AB 20 (2017) 104401 (2017) 67

Influence of incoherent scattering on planar channeling of high-energy negative particle beams in bent crystals

$$\alpha=\frac{L}{R}=\frac{\theta_c^2}{2\xi^2 R\left(\mathrm{erf}^{-1}\left(\frac{f}{1-\sqrt{\frac{R_c}{R}}}\right)\right)^2},\qquad \theta_c=\sqrt{\frac{2U_0}{E}}\left(1-\frac{R_c}{R}\right)$$

$$R=R_{\mathrm{opt}}\ \Rightarrow\ \frac{d\alpha}{dR}=0$$





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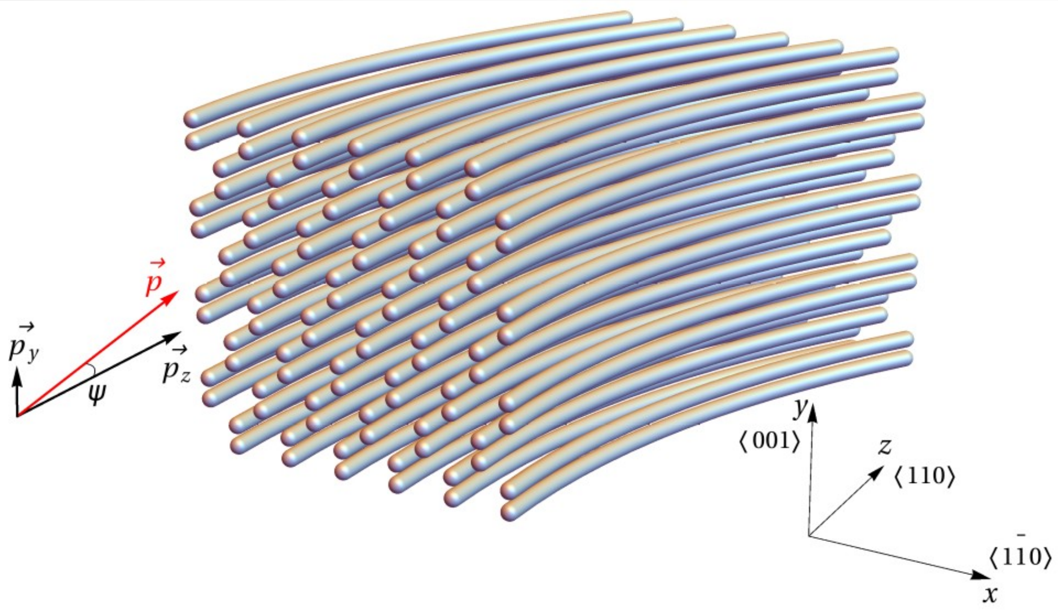
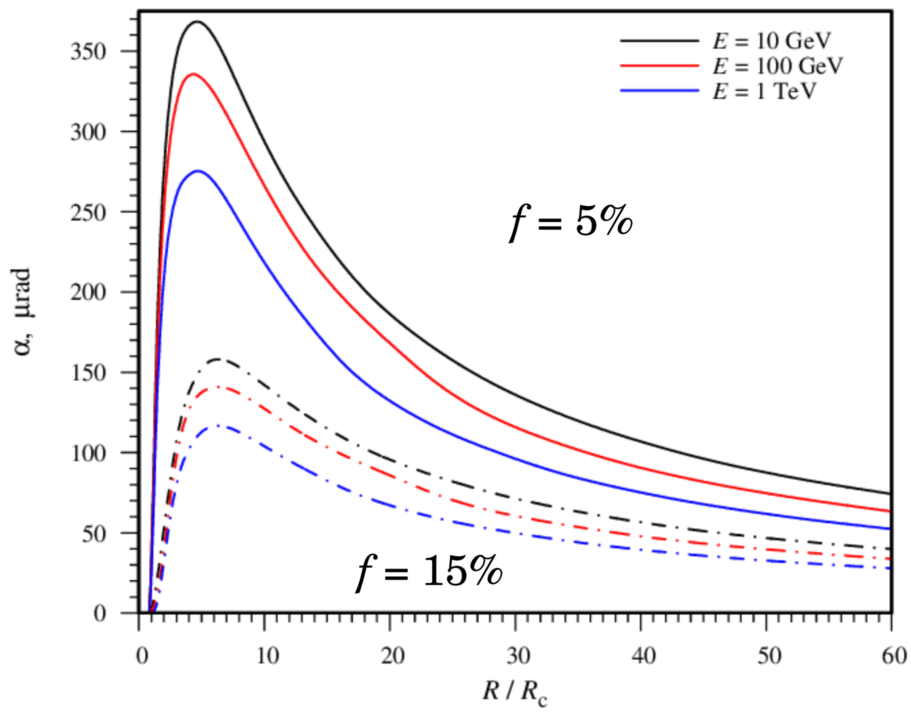
Influence of incoherent scattering on planar channeling of high-energy negative particle beams in bent crystals

π^-, Si (110)





I.V. Kirillin, Phys. Rev. AB 20 (2017) 104401 (2017) 67

Planar potential

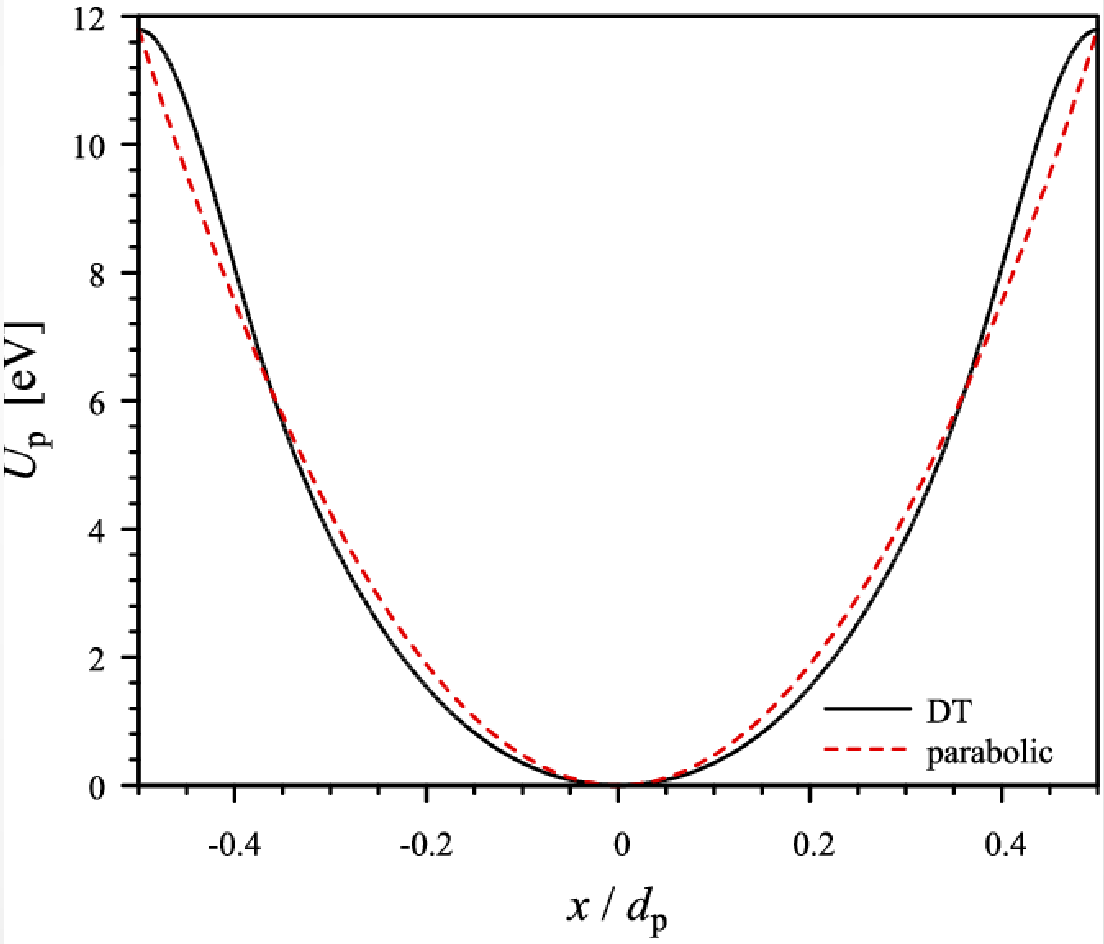
$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{d}{dx}U_p(x)$$

$$U_p(x) = U_0\left(\frac{2x}{d_p}\right)^2$$

$$\frac{d^2x}{dt^2} = -\frac{8c^2U_0}{Ed_p^2}x = -\frac{4c^2\theta_c^2}{d_p^2}x$$

$$x = C_1\cos(\omega_p t) + C_2\sin(\omega_p t)$$

$$\omega_p = \frac{2c\theta_c}{d_p}$$



Channeling radiation

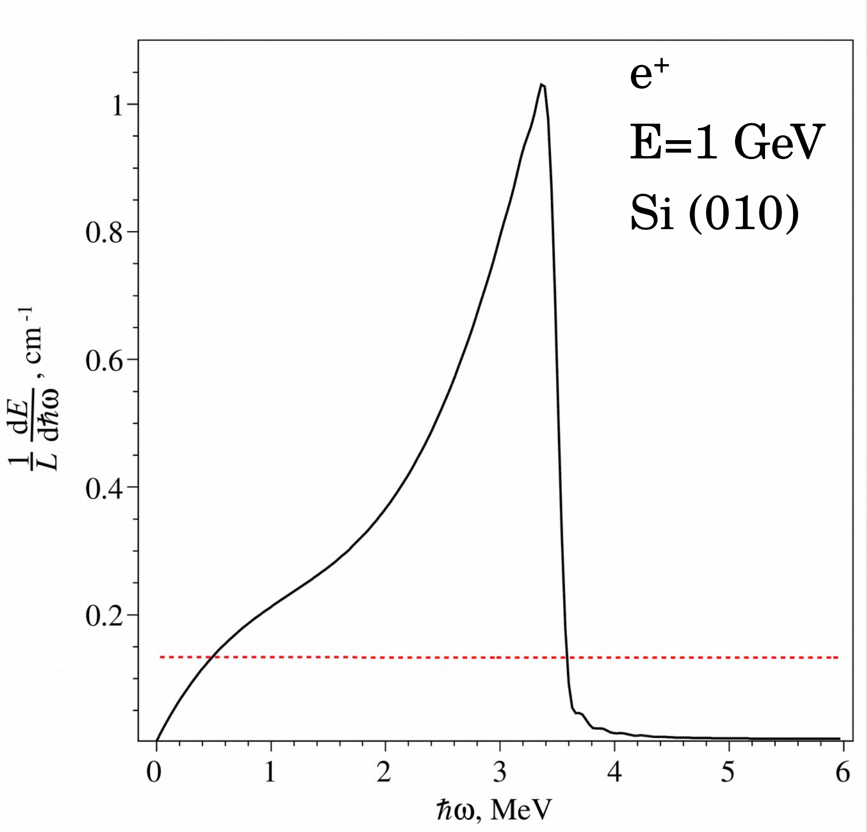
$$\frac{dE}{d\omega}=\frac{e^2\omega}{2\pi c^4}\int_\delta^\infty \frac{dq}{q^2}\left(1-2\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right)|\vec{W}(q)|^2$$

$$\vec{W}(q)=\int_0^T \dot{\vec{v}}\exp(iqct)\,dt\,,\quad \delta=\frac{\omega}{2c\gamma^2}$$

$$U_p(x)=U_0\left(\frac{2x}{d_p}\right)^2$$

$$\frac{dE}{d\omega}=A_1\omega_p^2\omega L\left(1-2\frac{\omega}{\omega_m}\left(1-\frac{\omega}{\omega_m}\right)\right)\Theta(\omega_m-\omega)$$

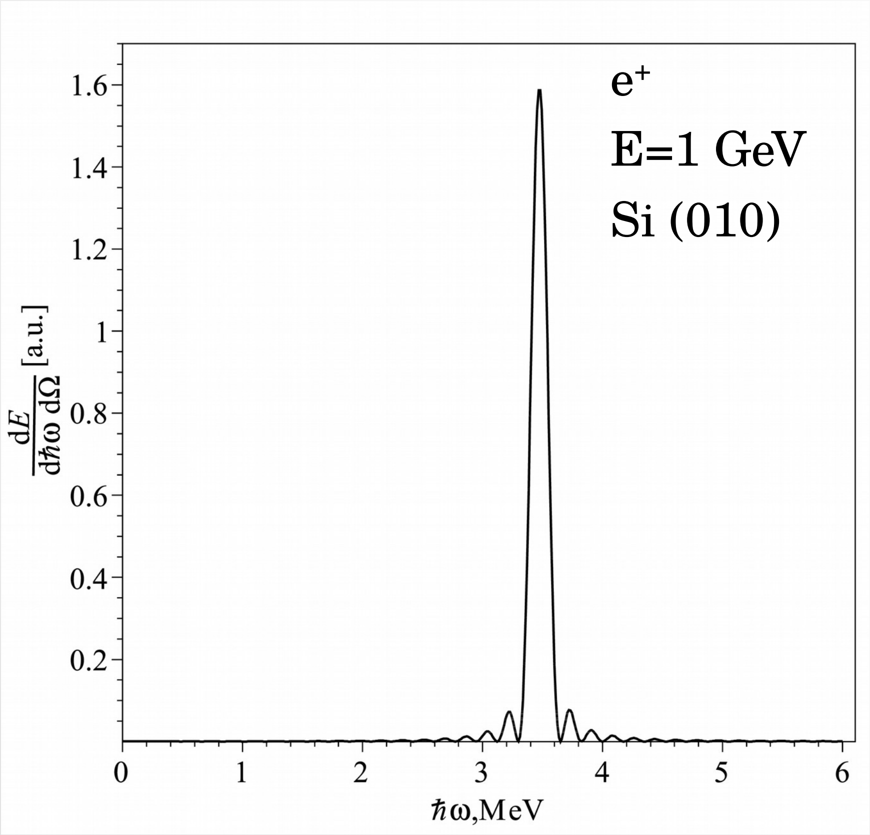
$$\omega_m=\frac{4\gamma^2 c\theta_c}{d_p}$$

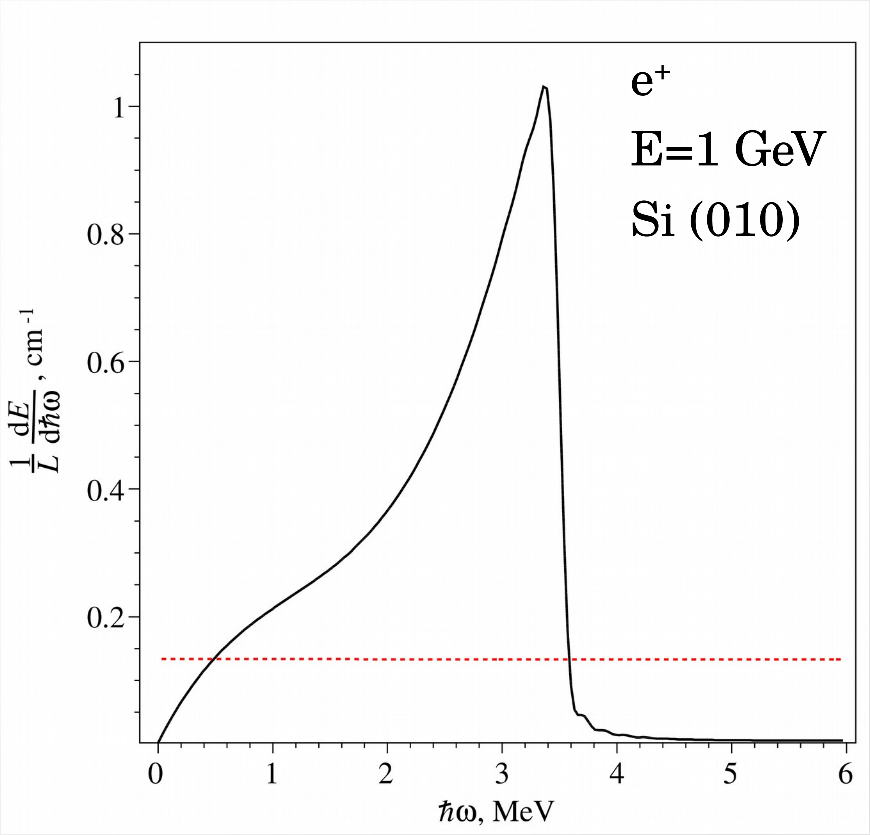


$$\hbar\omega_m\approx 3.42\,\mathrm{MeV}$$

Channeling radiation

$$\frac{dE}{d\omega\, d\Omega} = \frac{e^2\omega}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\, \frac{\vec{n}\times\left(\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right)}{\left(1-\vec{n}\vec{\beta}\right)^2}\exp\left(i\,\omega\left(t-\frac{\vec{n}\,\vec{r}}{c}\right)\right)\right|^2$$

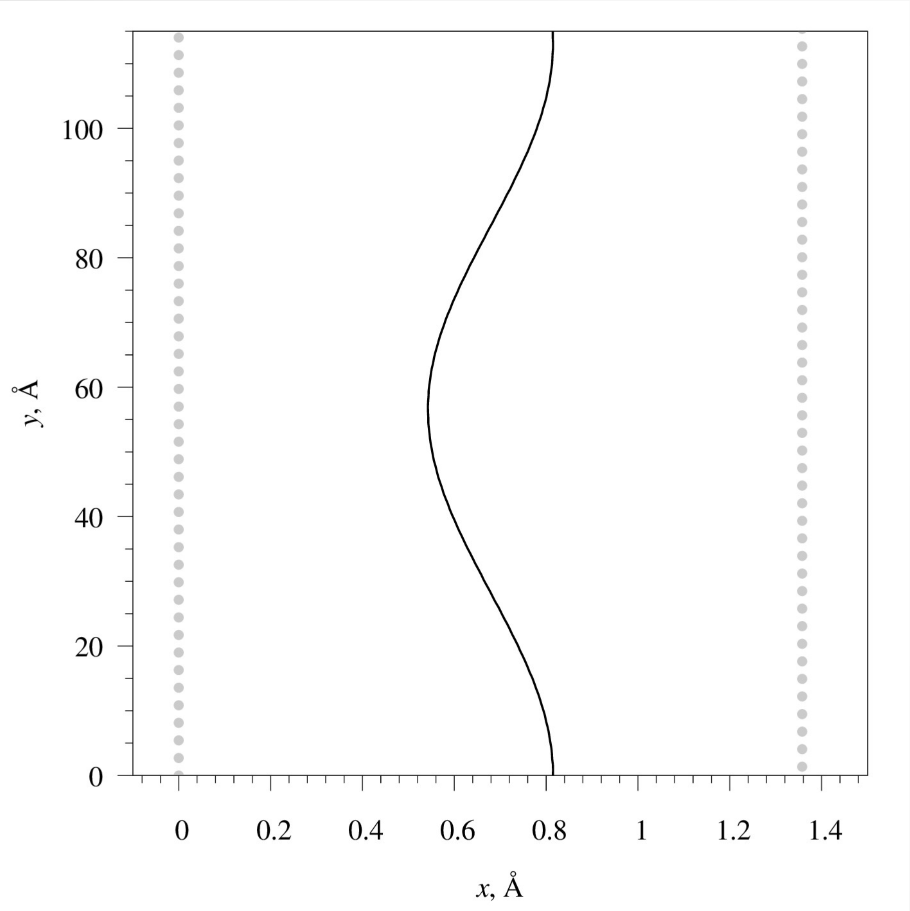




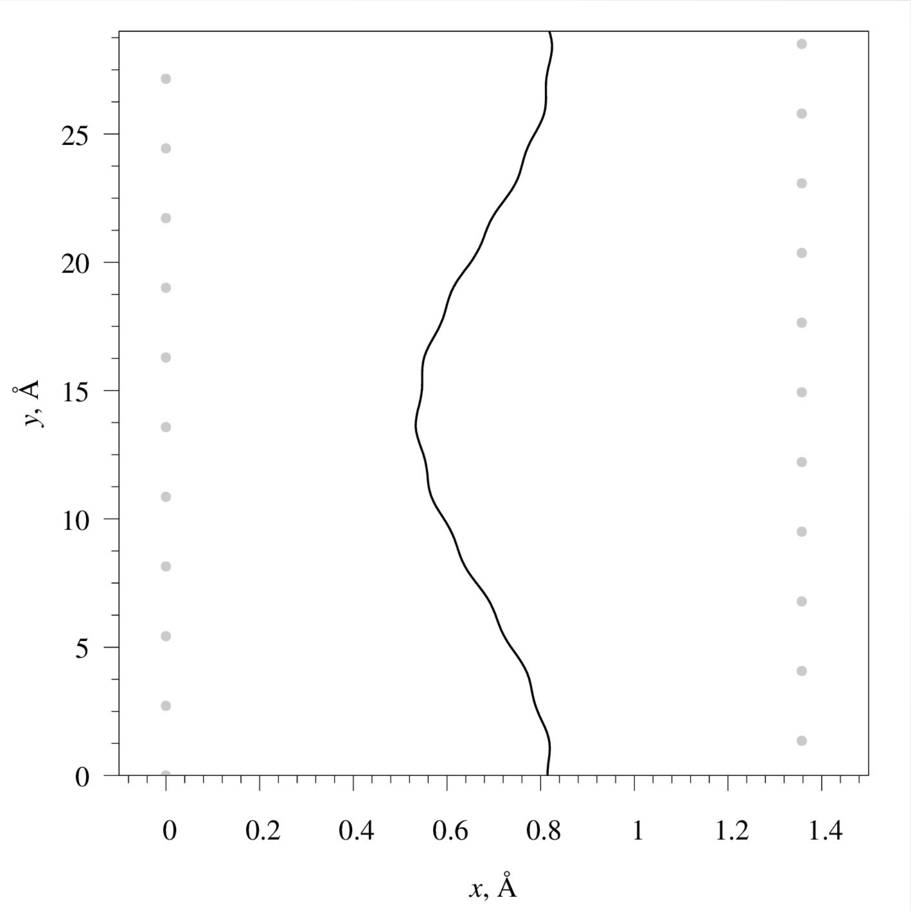
The effect of scattering on individual atomic strings

e^+, E=1 GeV, Si (010)

$\theta_{y,\mathrm{in}} = 10\,\psi_c$



$\theta_{y,\mathrm{in}} = 2.5\,\psi_c$

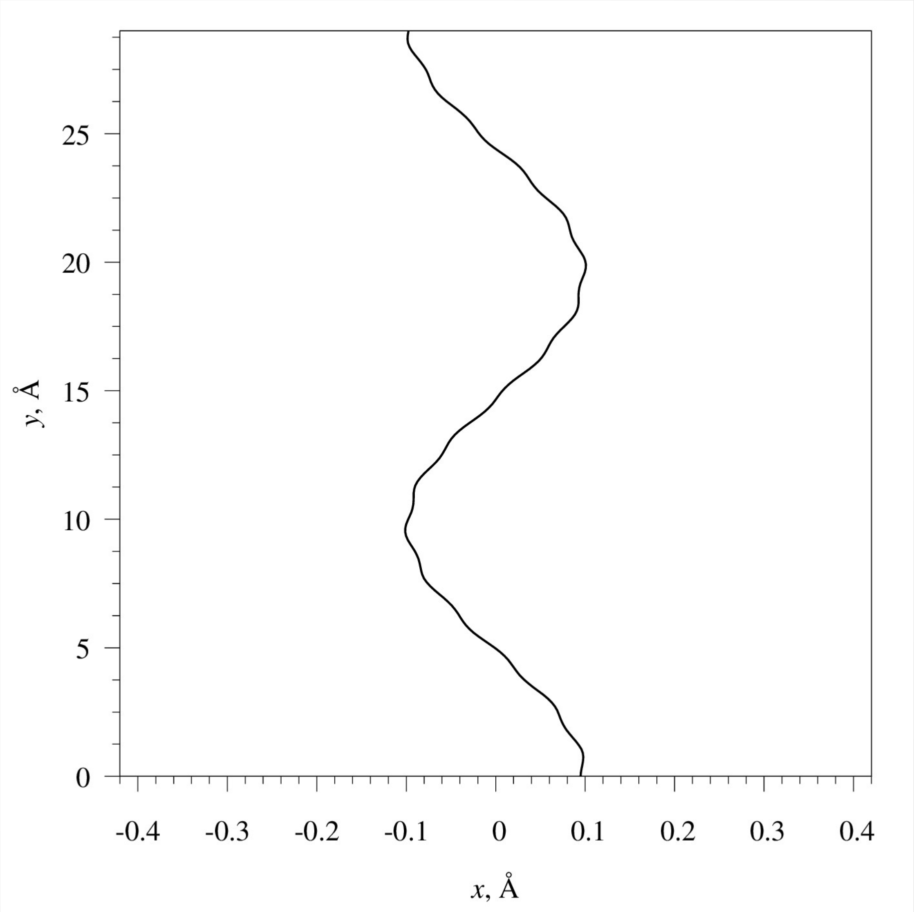


The effect of scattering on individual atomic strings

$$\frac{d^2x}{dt^2} = -\frac{4c^2\theta_c^2}{d_p^2}x + A\cos\left(2\pi\frac{v_y t}{d_s}\right), \qquad v_y \approx \theta_y c$$

$$x = C_1\cos(\omega_p t) + C_2\sin(\omega_p t) + \frac{A\cos(\omega_s t)}{\omega_p^2 - \omega_s^2},$$

$$\omega_s = \frac{2\pi c\theta_y}{d_s}$$



The effect of scattering on individual atomic strings

$$\frac{d^2x}{dt^2} = -\omega_p^2 x + A\cos(\omega_s t)$$

$$\frac{dE}{d\omega} = A_1 \omega_p^2 \omega L \left(1 - 2\frac{\omega}{\omega_{mp}}\left(1 - \frac{\omega}{\omega_{mp}}\right)\right)\Theta(\omega_{mp} - \omega) +$$

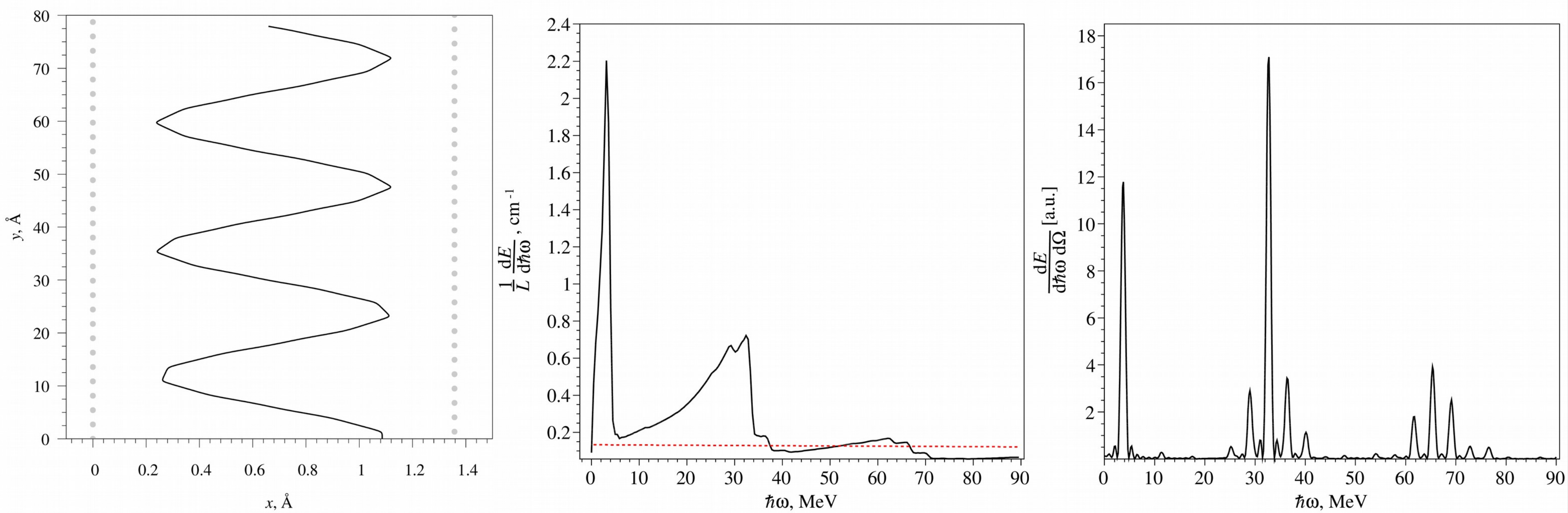
$$+ A_2 \frac{A^2}{(\omega_p^2 - \omega_s^2)^2}\omega_s^2 \omega L \left(1 - 2\frac{\omega}{\omega_{ms}}\left(1 - \frac{\omega}{\omega_{ms}}\right)\right)\Theta(\omega_{ms} - \omega), \quad \text{if} \quad \omega_s \neq \omega_p$$

$$\omega_{mp} = 2\gamma^2 \omega_p, \quad \omega_{ms} = 2\gamma^2 \omega_s, \quad \omega_{ms} \propto \theta_y$$

The effect of scattering on individual atomic strings

e^+, E=1 GeV, Si (010), L = 8.16 μm

$$\theta_{y,\mathrm{in}} = 2.5\,\psi_c \approx 0.96\,\mathrm{mrad}$$



$$\omega_{\mathrm{mp}} = 2\gamma^2 \omega_{\mathrm{p}} \approx 3.4\,\mathrm{MeV}, \quad \omega_{\mathrm{ms}} = 2\gamma^2 \omega_{\mathrm{s}} \approx 33.7\,\mathrm{MeV}$$

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Thank you for excellent school you make every year!

For lecturers:

Thank you for your interesting speeches and your work with students!

For students:

Thank you for your attention!