Understanding the nucleon structure: basic formalism, modern tools and open questions

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#### Part I

- Introduction
  - Motivation and scales
- Phenomenology
  - Elementary reactions
    - kinematics, crossing symmetry
    - Elastic scattering
    - Jlab and the GEp experiment
- Applications
  - Polarization
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#### Part II

- Phenomenology
  - Elementary reactions
    - Annihilation reactions
    - The PANDA experiment
- Form factors in annihilation and scattering: understanding how matter is formed







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- Phenomenology
  - Elementary reactions
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### SL & TL Form Factors







## The time-like region

$$p(p_1) + p(-p_2) \longrightarrow e^-(k_2) + e^+(-k_1)$$

$$e(k_1) + e^{-}(-k_2) \rightarrow p(-p_1) + p(p_2)$$

The measurement of the differential cross section

- At a fixed value of s
- For two different angles allows the separation of GE and GM.

TL equivalent of the Rosenbluth separation in SL region

- It is simpler in TL region, because a collider works at constant s and  $4\pi$  detectors allow to cover all angular range
- No individual determination of GE and GM has been done up to now
- Present and next future

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 $\cos^2 \tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t(\frac{t}{r} - M^2)} \to 1 +$ 



## Annihilation channel



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## The matrix element

$$p(p_1) + p(-p_2) \longrightarrow e^-(k_2) + e^+(-k_1)$$

The Matrix element:



$$\mathcal{M}=rac{e^2}{q^2}\overline{v}(k_2)\gamma_\mu u(k_1)\overline{u}(p_2)J_\mu v(p_1),$$

$$egin{aligned} J_{\mu} &= \left[F_1(q^2)\gamma_{\mu} - rac{\sigma_{\mu
u}q_{
u}}{2M_p}F_2(q^2)
ight] \ &= \left[F_1(q^2) + F_2(q^2)
ight]\gamma_{\mu} - rac{(-p_1+p_2)_{\mu}}{2M_p}F_2(q^2), \end{aligned}$$

In terms of Pauli  $\sigma$  matrices:

$$\vec{J} = \sqrt{q^2} \varphi_2^{\dagger} \left[ G_M(q^2) (\vec{\sigma} - \hat{\mathbf{p}} \vec{\sigma} \cdot \hat{\mathbf{p}}) + \frac{1}{\sqrt{\tau}} G_E(q^2) \hat{\mathbf{p}} \vec{\sigma} \cdot \hat{\mathbf{p}} \right] \varphi_1,$$

 $ec{L}=\sqrt{q^2}arphi_2^\dagger(ec{\sigma}-\hat{f k}ec{\sigma}\cdot\hat{f k})arphi_1,$ 

- Unpolarized leptons
- Threshold:  $G_E(q^2=4M^2)=G_M(q^2=4M_p^2)$
- $\hat{\vec{p}}\vec{\sigma}\cdot\hat{\vec{p}}$  annihilation from *D-state*





### Annihilation Cross Section (1)

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{|\overline{\mathcal{M}}|^2}{64\pi^2 q^2} \frac{k}{p}, \ k = \frac{\sqrt{q^2}}{2}, \ p = \sqrt{\frac{q^2}{4} - m^2}$$

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{4} \frac{e^4}{q^4} L_{ab} J_{ab}, \ L_{ab} = L_a L_b^*, \ J_{ab} = J_a J_b^*. \\ \overline{L_{ab}} &= \overline{L_a L_b^*} \sim Tr(\sigma_a - \hat{k}_a \vec{\sigma} \cdot \vec{k})(\sigma_b - \hat{k}_b \vec{\sigma} \cdot \vec{k}) = 2(\delta_{ab} - k_a k_b) \end{aligned}$$

as  $\vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{p} = \vec{p} \cdot \vec{p} = p^2 = 1$  (unit vectors)  $p_a \vec{\sigma} \cdot \vec{p} \sigma_b = p_a (\vec{p} \cdot \hat{b} + i\vec{\sigma} \cdot \vec{p} \times \hat{b})$  and Tr of one  $\sigma$  vanish.  $Tr \vec{\sigma} \cdot \vec{a} \vec{\sigma} \cdot \vec{b} \vec{\sigma} \cdot \vec{c} = i\vec{a} \cdot \vec{b} \times \vec{c}.$ 

Hadron tensor  $J_{ab}$ : the product gives four terms. Let us classify along FFs.

$$ec{J} = \sqrt{q^2} arphi_2^\dagger \left[ G_M(q^2) (ec{\sigma} - \hat{\mathbf{p}} ec{\sigma} \cdot \hat{\mathbf{p}}) + rac{1}{\sqrt{ au}} G_E(q^2) \hat{\mathbf{p}} ec{\sigma} \cdot \hat{\mathbf{p}} 
ight] arphi_1,$$



### Annihilation Cross Section (2)

$$egin{aligned} ec{J} = \sqrt{q^2}arphi_2^\dagger \left[ G_M(q^2) (ec{\sigma} - \hat{\mathbf{p}}ec{\sigma} \cdot \hat{\mathbf{p}}) + rac{1}{\sqrt{ au}} G_E(q^2) \hat{\mathbf{p}}ec{\sigma} \cdot \hat{\mathbf{p}} 
ight] arphi_1, \end{aligned}$$

$$1)|G_{M}|^{2} : \frac{1}{2}Tr(\sigma_{a} - p_{a}\vec{\sigma} \cdot \vec{p})(\sigma_{b} - p_{b}\sigma \cdot p)$$

$$\rightarrow \sigma_{a}\sigma_{b} - p_{a}\vec{\sigma} \cdot \vec{p}\sigma_{b} - \sigma_{a}p_{b}\vec{\sigma} \cdot \vec{p} + p_{a}p_{b}\vec{\sigma} \cdot \vec{p}\vec{\sigma} \cdot \vec{p}$$

$$= \delta_{ab} - p_{a}p_{b} - p_{b}p_{a} + p_{a}p_{b} = \delta_{ab} - p_{a}p_{b}$$

2)  $|G_E|^2 : \frac{1}{\tau} p_a \vec{\sigma} \cdot \vec{p} p_b \vec{\sigma} \cdot \vec{p} = \frac{1}{\tau} p_a p_b$ 3) No interference terms :

$$\begin{aligned} G_E G_M^* &: \frac{1}{2} Tr[p_a \vec{\sigma} \cdot \vec{p} (\sigma_b - p_b \vec{\sigma} \cdot \vec{p})] \\ & \rightarrow \frac{1}{\sqrt{\tau}} [p_a \vec{\sigma} \cdot \vec{p} \sigma \cdot \hat{b} - p_a p_b \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{p}] \\ &= (p_a p_b - p_a p_b) = 0 \end{aligned}$$

4) Similarly  $G_M G_E^* : \frac{1}{2} Tr \frac{1}{\tau} (\sigma_a - p_a \sigma \cdot p) p_b \vec{\sigma} \cdot \vec{p} = 0.$ 





### Annihilation Cross Section (3)

We took into account the properties of  $\sigma$  matrices :  $\vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{p} = p^2 = 1$ ,  $Tr\vec{\sigma} \cdot \vec{a}\vec{\sigma} \cdot \vec{b}\vec{\sigma} \cdot \vec{c} = i\vec{a} \cdot \vec{b} \times \vec{c}$ .





## Annihilation Cross Section (4)

$$\overline{p}(p_1) + p(-p_2) \rightarrow e^{-}(k_2) + e^{+}(-k_1)$$

The differential cross section

$$\begin{pmatrix} \frac{d\sigma}{d\Omega} \end{pmatrix}_0 = \mathcal{N} \left[ (1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \qquad \mathcal{N} = \frac{\alpha^2}{4\sqrt{q^2(q^2 - 4m^2)}}.$$

$$Magnetic \qquad Electric \qquad \alpha = e^2/(4\pi) \simeq 1/137.$$

The angular Asymmetry

$$\begin{pmatrix} \frac{d\sigma}{d\Omega} \end{pmatrix}_{0} = \sigma_{0} \left[ 1 + \mathcal{A} \cos^{2} \theta \right] |, \qquad \mathcal{A} = \frac{\tau |G_{M}|^{2} - |G_{E}|^{2}}{\tau |G_{M}|^{2} + |G_{E}|^{2}} = \frac{\tau - \mathcal{R}^{2}}{\tau + \mathcal{R}^{2}}.$$

$$\sigma_{0} = \frac{\alpha^{2}}{4q^{2}} \sqrt{\frac{\tau}{\tau - 1}} \left( |G_{M}|^{2} + \frac{1}{\tau}|G_{E}|^{2} \right) \qquad \mathcal{R} = |G_{E}|/|G_{M}|$$

The total cross section

$$\sigma(q^2) = \mathcal{N}\frac{8}{3}\pi \left[2|G_M|^2 + \frac{1}{\tau}|G_E|^2\right].$$



## Polarized Antiprotons (1)

 $\vec{P}_1$  and  $\vec{P}_2$  : polarizations of the colliding antiproton and proton :

$$\begin{pmatrix} \frac{d\sigma}{d\Omega} \end{pmatrix}_0 (\vec{P}_1, \vec{P}_2) = \left( \frac{d\sigma}{d\Omega} \right)_0 [1 + A_y (P_{1y} + P_{2y}) + A_{xx} P_{1x} P_{2x} + A_{yy} P_{1y} P_{2y} + A_{zz} P_{1z} P_{2z} + A_{xz} (P_{1x} P_{2z} + P_{1z} P_{2x})]$$

where  $A_i$  and  $A_{ij}$  (i, j = x, y, z) are the analyzing powers and correlation coefficients, and depend on the nucleon FFs. The polarized hadronic tensor :

$$W_{ab}(\vec{P}_1,\vec{P}_2) = \frac{1}{2} \operatorname{Tr} J_a \vec{\sigma} \cdot \vec{P}_1 J_b^* \vec{\sigma} \cdot \vec{P}_2.$$

The cross section with unpolarized electrons is proportional to  $L_{ab}\overline{W_{ab}}$ .





## Polarized antiproton beam (2)

$$\begin{pmatrix} d\sigma \\ d\Omega \end{pmatrix}_{0} \vec{A}_{1} \sim -L_{ab} \frac{1}{4} \operatorname{Tr} J_{a} \vec{\sigma} J_{b}^{*} = \\ [(\sigma_{a} - p_{a} \vec{\sigma} \cdot \vec{p}) G_{M} + \frac{1}{\tau} G_{E} p_{a} \vec{\sigma} \cdot \vec{p}] (-\vec{\sigma} \cdot \vec{P}_{1}) \\ [(\sigma_{b} - p_{b} \vec{\sigma} \cdot \vec{p}) G_{M}^{*} + \frac{1}{\tau} G_{E}^{*} p_{b} \vec{\sigma} \cdot \vec{p}] (\delta_{ab} - k_{a} K_{b})$$

Note :  $Tr\vec{\sigma} \cdot \vec{a}\vec{\sigma} \cdot \vec{b}\vec{\sigma} \cdot \vec{c} = i\vec{a} \cdot \vec{b} \times \vec{c} = ib \cdot \vec{c} \times \vec{a} = i\vec{c} \cdot \vec{a} \times \vec{b}$ For antiparticles we remember a global general sign :  $(-\vec{\sigma} \cdot \vec{P}_1)$ 





## Polarized antiproton beam $|G_M|^2(3)$

 $|G_{M}|^{2}$  :

$$[1]: (\sigma_{a} - p_{a}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}(\sigma_{b} - p_{b}\vec{\sigma} \cdot \vec{p})\delta_{ab} - [2]: (\sigma_{a} - p_{a}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}(\sigma_{b} - p_{b}\vec{\sigma} \cdot \vec{p})\hat{k}_{a}\hat{k}_{b}$$

$$[1]: \sigma_{a}\vec{\sigma} \cdot \vec{P}_{1}\sigma_{a} - p_{a}\vec{\sigma} \cdot \vec{p}\vec{\sigma} \cdot \vec{P}_{1}\sigma_{a} - \sigma_{a}\vec{\sigma} \cdot \vec{P}_{1}p_{a}\vec{\sigma} \cdot \vec{p} + p_{a}\vec{\sigma} \cdot \vec{p}\vec{\sigma} \cdot \vec{P}_{1}p_{a}\vec{\sigma} \cdot \vec{p}$$

$$= -p_{a}(a \cdot P_{1} \times \vec{p} + p \cdot P_{1} \times \vec{a}) + p_{a}^{2}\vec{\sigma} \cdot \vec{P}_{1} = 0$$

$$[2]: (\vec{\sigma} \cdot \vec{k} - \vec{p} \cdot \vec{k}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}(\vec{\sigma} \cdot \vec{k} - \vec{p} \cdot \vec{k}\vec{\sigma} \cdot \vec{p}) + \vec{p} \cdot \vec{k}\vec{\sigma} \cdot \vec{p} - \vec{k} \cdot \vec{k} - \vec{k} - \vec{k}\vec{k} \cdot \vec{k} - \vec{k} - \vec{k} - \vec{k}\vec{k} \cdot \vec{k} - \vec{k} - \vec{k} - \vec{k} - \vec{k}\vec{k} \cdot \vec{k} - \vec{k} - \vec{k} - \vec{k} \cdot \vec{k} - \vec{k$$

$$|\mathbf{G}_{\mathbf{E}}|^{2}: \quad \frac{1}{\tau} \left[ p_{\mathbf{a}} \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_{1} p_{\mathbf{a}} \vec{\sigma} \cdot \vec{p} - (\vec{p} \cdot \vec{k})^{2} \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_{1} \vec{\sigma} \cdot \vec{p} \right] = 0$$



## Polarized antiproton beam $G_E G_M^*(4)$

$$G_{M}G^{*}{}_{E}: \qquad \frac{1}{\sqrt{\tau}}[(\vec{\sigma}_{a} - p_{a}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}p_{b}\vec{\sigma} \cdot \vec{p}](\delta_{ab} - k_{a}k_{b})$$

$$= \frac{1}{\sqrt{\tau}}[(\vec{\sigma}_{a} - p_{a}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}p_{a}\vec{\sigma} \cdot \vec{p} - (\vec{\sigma} \cdot \vec{k} - \vec{p} \cdot \vec{k}\vec{\sigma} \cdot \vec{p})\vec{\sigma} \cdot \vec{P}_{1}\vec{p} \cdot \vec{k}\vec{\sigma} \cdot \vec{p}]$$

Explicitly each component:

$$\begin{aligned} &\frac{1}{\tau} [(\sigma_x \vec{\sigma} \cdot \vec{P}_1 p_x \sigma_z + \sigma_y \vec{\sigma} \cdot \vec{P}_1 p_y \sigma_z) \\ &- (\sigma_x \sin \theta + \sigma_z \cos \theta - \sigma_z \cos \theta) \vec{\sigma} \cdot \vec{P}_1 \cos \theta \sigma_z] \\ &= -\sigma_x \sin \theta \cos \theta \vec{\sigma} \cdot \vec{P}_1 \sigma_z = -i \sin \theta \cos \theta P_{1y} \end{aligned}$$





## Polarized antiproton beam $G_E G_M^*(5)$

$$G_E G^*_M$$
:

$$\begin{split} & [p_a \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_1 (\sigma_b - p_b \vec{\sigma} \cdot \vec{p})] (\delta_{ab} - k_a k_b) \\ = & [p_a \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_1 \vec{\sigma}_a - p_a \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_1 p_a \vec{\sigma} \cdot \vec{p} - \\ & \vec{p} \cdot \vec{k} \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_1 \vec{\sigma} \cdot \vec{k} - (\vec{p} \cdot \vec{k})^2 \vec{\sigma} \cdot \vec{p} \vec{\sigma} \cdot \vec{P}_1 \vec{\sigma} \cdot \vec{p} \\ = & i [p_a \vec{a} \cdot \vec{p} \times \vec{P}_1 - \cos \theta \vec{p} \cdot \vec{P}_1 \times \vec{k}] \end{split}$$

$$\vec{a} \cdot \vec{p} \times \vec{P}_1 \to p_x = p_y = 0; z \cdot p_z \times P_1 = 0$$
$$\begin{pmatrix} p & 0 & 0 & 1 \\ P & P_{1x} & P_{1y} & P_{1z} \\ k & \sin \theta & 0 & \cos \theta \end{pmatrix}$$
$$G_E G_M^* \to \frac{i}{\sqrt{\tau}} \cos \theta \sin \theta P_{1y}$$





## Single spin observables

#### When the beam is polarized

$$\left(rac{d\sigma}{d\Omega}
ight)_0 A_{1,y} = -rac{i\mathcal{N}}{\sqrt{ au}}\sin heta\cos heta[G_MG_E^* - G_EG_M^*] = rac{\mathcal{N}}{\sqrt{ au}}\sin2 heta Im(G_MG_E^*).$$

#### When the target is polarized

$$\left(\frac{d\sigma}{d\Omega}\right)_{0}\vec{A}_{2} = L_{ab}\frac{1}{4}TrJ_{a}J_{b}^{*}\vec{\sigma}. \qquad \vec{A}_{2} = \vec{A}_{1} = \vec{A}.$$



#### Unitarity

In TL region single spin observables do not vanish: FFs are complex (q<sup>2</sup>>0). *Final state interaction* 





## Double spin polarization observables

$$\left(rac{d\sigma}{d\Omega}
ight)_{0}A_{ab}=-rac{1}{4}L_{mn}TrJ_{m}\sigma_{a}J_{n}^{\dagger}\sigma_{b},$$

#### The nonzero components are:

$$\begin{split} & \left(\frac{d\sigma}{d\Omega}\right)_{0} A_{xx} &= \sin^{2}\theta \left(|G_{M}|^{2} + \frac{1}{\tau}|G_{E}|^{2}\right)\mathcal{N}, \\ & \left(\frac{d\sigma}{d\Omega}\right)_{0} A_{yy} &= -\sin^{2}\theta \left(|G_{M}|^{2} - \frac{1}{\tau}|G_{E}|^{2}\right)\mathcal{N}, \\ & \left(\frac{d\sigma}{d\Omega}\right)_{0} A_{zz} &= \left[(1 + \cos^{2}\theta)|G_{M}|^{2} - \frac{1}{\tau}\sin^{2}\theta|G_{E}|^{2}\right]\mathcal{N}, \\ & \left(\frac{d\sigma}{d\Omega}\right)_{0} A_{xz} &= \left(\frac{d\sigma}{d\Omega}\right)_{0} A_{zx} = \frac{1}{\sqrt{\tau}}\sin 2\theta ReG_{E}G_{M}^{*}\mathcal{N}. \end{split}$$

#### Relative phase: Ay, Axz



### Facility for Antiproton and Ion Research (Darmstadt/Germany)



CBM Heavy Ions

1111

### PANDA Anti-protons

## All physics communities are represented



FAIR Facility for Antiproton and Ion Research in Europe GmbH





## Antiprotons at FAIR

#### http://www.fair-center.eu/

#### https://panda.gsi.de

#### Parameters of HESR

- Injection of pbar at 3.7 GeV
- Slow synchrotron
- Momentum 1.5-15 GeV/c
- Luminosity 10<sup>31</sup> cm<sup>-2</sup>s<sup>-1</sup> (up to  $L_{peak} \sim 2 \times 10^{32}$  cm<sup>-2</sup>s<sup>-1</sup>)
- Beam cooling



#### Pbar production

Proton Linac 70 MeV
Accelerate p in SIS18/SIS100
Produce pbar on NI/Cu target
Collect pbar in CR
Storage in HESR



## Antiproton facilities

Experiment	Years	Intensity	Momentum range	$\Delta p/p$
		$\bar{p}/s$	[GeV/c]	
CERN -LEAR	1983-1996	$2\cdot 10^6$	0.06-1.94	$10^{-3}$
FermiLab	1985-2011	$2\cdot 10^6$	<8.9	$10^{-4}$
45% polarized $\bar{p}$		$10^{4}$	(Low energy beams)	
PANDA		$2\cdot 10^7$	1.5 - 15	$10^{-5}$

#### Panda will have:

- Better luminosity
- Better beam momentum resolution
- Better detector (coverage, PID, magnetic field..)



### Proton-Antiproton Annihilation



- Formation:
  - -> (precision physics)



J = 0, 2, ... C = +



## J = 1, ... C = -

- q-qbar annihilate into gluons
- gluon-rich environment
- all quantum numbers allowed by qqbar

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## Search for glueballs, hybrids

Very precise scan of a resonance in formation mode: depends on HESR beam momentum resolution Dp/p~2x10<sup>-5</sup>



<u>Appearance of a resonance in production mode and disappearance in</u> formation mode sign its exotic nature





# Hadron Physics











### START/ FULL SETUP





## Hadron Electromagnetic Form factors



### Antiprotons at FAIR





## Threshold physics

#### VEPPII, Novosibirsk

1800

1850

#### Beijing, BEPC2, BES3 400 $e^+e^- \rightarrow \Lambda_c^+ \overline{\Lambda}_c^-$ BESIII data 300 Belle data BESIII fit PHSP model σ (pb) Threshold 200 Λ 100 **BESIII** 0 4.56 4.57 4.58 4.59 4.6 (GeV) RPC:8 RPC: 9 lavers layers Electro Magnetic Calorimeter σ (nb) SC Solenoid n 30s0=0.83 2012 Barrel 2011 cos0=0.90 Ρ ToF FENICE Endcap 1 cos0=0.93 ToF SC Quadrupole E 0 1900 1950

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29

1950

1900

2000 2E<sub>b</sub> (MeV)

# The Time-like Region



Expected QCD scaling  $(q^2)^2$ 

$$|F_{scaling}(q^2)| = \frac{\mathcal{A}}{(q^2)^2 \log^2(q^2/\Lambda^2)}$$



# The Time-like Region



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# Oscillations : regular pattern in P<sub>Lab</sub>

The relevant variable is  $p_{Lab}$  associated to the relative motion of the final hadrons.



A. Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015)



# Oscillations : regular pattern in P<sub>Lab</sub>



iated to the relative

$$p) \equiv A \exp(-Bp) \cos(Cp + D).$$

	$B\pm \Delta B$	$C\pm\Delta C$	$D \pm \Delta D$	$\chi^2/n.d.f$
	$[GeV]^{-1}$	$[GeV]^{-1}$		
)1	$0.7\pm0.2$	$5.5\pm0.2$	$0.03\pm0.3$	1.2

Il perturbationB: dampingfmD=0: maximum at p=0

le oscillatory behaviour I number of coherent sources

A.Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015), PRC 93, 035201 (2016)



# Fourier Transform



- Rescattering processes
- Large imaginary part
- Related to the time evolution of the charge density? (E.A. Kuraev, E. T.-G., A. Dbeyssi, PLB712 (2012) 240)
- Consequences for the SL region?
- Data from BESIII confirm the structures
- Expected from PANDA



### VMD: Iachello, Jakson and Landé (1973)

#### Isoscalar and isovector FFs



$$\begin{split} F_1^s(Q^2) \ &= \ \frac{g(Q^2)}{2} \left[ (1 - \beta_\omega - \beta_\phi) + \beta_\omega \frac{\mu_\omega^2}{\mu_\omega^2 + Q^2} + \beta_\phi \frac{\mu_\phi^2}{\mu_\phi^2 + Q^2} \right], \\ F_1^v(Q^2) \ &= \ \frac{g(Q^2)}{2} \left[ (1 - \beta_\rho) + \beta_\rho \frac{\mu_\rho^2 + 8\Gamma_\rho \mu_\pi / \pi}{(\mu_\rho^2 + Q^2) + (4\mu_\pi^2 + Q^2)\Gamma_\rho \alpha(Q^2) / \mu_\pi} \right], \\ F_2^s(Q^2) \ &= \ \frac{g(Q^2)}{2} \left[ (\mu_p + \mu_n - 1 - \alpha_\phi) \frac{\mu_\omega^2}{\mu_\omega^2 + Q^2} + \alpha_\phi \frac{\mu_\phi^2}{\mu_\phi^2 + Q^2} \right], \\ F_2^v(Q^2) \ &= \ \frac{g(Q^2)}{2} \left[ (\mu_p - \mu_n - 1) \frac{\mu_\rho^2 + 8\Gamma_\rho \mu_\pi / \pi}{(\mu_\rho^2 + Q^2) + (4\mu_\pi^2 + Q^2)\Gamma_\rho \alpha(Q^2) / \mu_\pi} \right], \end{split}$$

 $\sqrt{*}$ 

ω,ρ,φ

$$g(Q^2) = \frac{1}{(1+\gamma e^{i\theta}Q^2)^2}$$
$$\alpha(Q^2) = \frac{2}{\pi} \sqrt{\frac{Q^2+4\mu_\pi^2}{Q^2}} ln \left[ \frac{\sqrt{(Q^2+4\mu_\pi^2)}+\sqrt{Q^2}}{2\mu_\pi} \right]$$

$$2F_i^p = F_i^s + F_i^v,$$
  
$$2F_i^n = F_i^s - F_i^v.$$





# The nucleon



3 valence quarks and a neutral sea of  $q\overline{q}$  pairs

antisymmetric state of colored quarks

 $|p \rangle \sim \epsilon_{ijk} |u^{i}u^{j}d^{k} \rangle$  $|n \rangle \sim \epsilon_{ijk} |u^{i}d^{j}d^{k} \rangle$ 

New assumption :

...does not hold in the spatial center of the nucleon: the center of the nucleon *is electrically neutral*, due to strong gluonic field

E.A. Kuraev, E. T-G, A. Dbeyssi, Phys.Lett. B712 (2012) 240







# **Definition of TL-SL Form Factors**

$$F(q) = \int d^4x e^{iqx} F(x).$$

$$F_{SL,Breit}(q) = \int d^3 \vec{x} \ e^{-i\vec{q}\cdot\vec{x}} \int dt F(t,\vec{x}) \equiv \int d^3 \vec{x} \ e^{-i\vec{q}\cdot\vec{x}} \rho(|\vec{x}|),$$
  
$$\rho(|\vec{x}|) = \int dt F(t,\vec{x}).$$

$$F_{TL,CM}(q) = \int dt \ e^{iqt} \int d^3 \vec{x} F(t, \vec{x}) \equiv \int dt \ e^{iqt} R(t),$$
$$R(t) = \int d^3 \vec{x} F(t, \vec{x}).$$

 $(\vec{x})$  and R(t), represent projections of the same distribution in orthogonal subspaces

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# Charge: photon-charge coupling



Fourier transform of a stationary charge and current distribution



Amplitude for creating charge-anticharge pairs at time t. Charge distribution => distribution in time of







# Conclusion - Discussion



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# (1) Equivalent forms $J_{\mu}$

$$J_{m \mu} \, = \left[F_1(q^2) + F_2(q^2)
ight] \gamma_{\mu} - rac{(-p_1+p_2)_{\mu}}{2M_p}F_2(q^2),$$

$$\begin{split} J_{\mu} &\rightarrow \varphi_{2} \tilde{J}_{\mu} \varphi_{1} \\ J_{\mu} &= (F_{1} + F_{2}) \left( \chi_{2}, \ -\frac{\vec{\sigma} \cdot (-\vec{p})}{E + m} \chi_{2} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \left( \begin{array}{cc} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{array} \right) \left( \begin{array}{cc} \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \chi_{1} \\ \chi_{1} \end{array} \right) \\ &+ \left( \chi_{2}, \ \frac{\vec{\sigma} \cdot (-\vec{p})}{E_{1} + m} \chi_{2} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \frac{2\vec{p}}{2m} F_{2} \left( \begin{array}{cc} \frac{\vec{\sigma} \cdot \vec{p}}{E_{1} + m} \chi_{1} \\ \chi_{1} \end{array} \right) \\ &= (F_{1} + F_{2}) \left( \chi_{2}, \ \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \chi_{2} \right) \left( \begin{array}{cc} -\vec{\sigma} \frac{\vec{\sigma} \chi_{1}}{\vec{\sigma} \cdot \vec{p}} \\ -\vec{\sigma} \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \chi_{1} \end{array} \right) + \frac{\vec{p}}{m} F_{2} \chi_{2} \left( \frac{\vec{\sigma} \cdot \vec{p}}{E + m} + \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \right) \chi_{1} \\ &= (F_{1} + F_{2}) \left[ \vec{\sigma} - \frac{1}{(E + m)^{2}} \vec{\sigma} \cdot \vec{p} \vec{\sigma} \vec{\sigma} \cdot \vec{p} \right] + \frac{2\vec{p}}{m} F_{2} \chi_{2} \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \chi_{1} \end{split}$$

 $(E + M_p)$  Global factor





Properties of Pauli  $\sigma$  matrices:

$$(2\hat{\vec{p}}-\vec{\sigma}\vec{\sigma}\cdot\hat{\vec{p}})\vec{\sigma}\cdot\hat{\vec{p}}=2\hat{\vec{p}}\vec{\sigma}\cdot\hat{\vec{p}}-\vec{\sigma}$$

$$J_{\mu} = (F_{1} + F_{2}) \left( \vec{\sigma} - 2\frac{\vec{E} - m}{\vec{E} + m} \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}} + \frac{\vec{E} - m}{\vec{E} + m} \vec{\sigma} \right) + \frac{2(\vec{E} - m)}{m} F_{2} \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}}$$

$$= (F_{1} + F_{2}) \left( \vec{\sigma} + \frac{\vec{E} - m}{\vec{E} + m} \vec{\sigma} \right) - 2 \left[ (F_{1} + F_{2}) \frac{\vec{E} - m}{\vec{E} + m} - \frac{\vec{E} - m}{m} F_{2} \right] \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}}$$

$$= \frac{2E}{\vec{E} + m} (F_{1} + F_{2}) \vec{\sigma} - \frac{2(\vec{E} - m)}{m(\vec{E} + m)} [mF_{1} + mF_{2} - \vec{E}F_{2} - mF_{2}] \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}}$$

$$= \frac{2E}{\vec{E} + m} (F_{1} + F_{2}) \vec{\sigma} - 2E(F_{1} + F_{2}) \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}} + 2m \left( F_{1} + \frac{\vec{E}^{2}}{m^{2}} F_{2} \right)$$

$$= \frac{2E}{\vec{E} + m} \left[ G_{M} (\vec{\sigma} - \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}}) \right] + 2m G_{E} \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}}$$

Finally (reminding the global factor (E + m)):

$$J = 2E \left[ G_M \left( \vec{\sigma} - \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}} \right) + \frac{1}{\sqrt{\tau}} G_E \hat{\vec{p}} \vec{\sigma} \cdot \hat{\vec{p}} \right]$$





# The proton radius





Egle Tomasi-Gustafsson



## Root mean square radius

$$F(q) = \frac{\int_{\Omega} d^3 \vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_{\Omega} d^3 \vec{x} \rho(\vec{x})}.$$

$$< r_c^2 >= \frac{\int_0^\infty x^4 \rho(x) dx}{\int_0^\infty x^2 \rho(x) dx}.$$

Expanding in Taylor series:

$$F(q) \sim 1 - \frac{1}{6}q^2 < r_c^2 > +O(q^2),$$

$$\langle r_{E/M}^2 \rangle = - \frac{6\hbar^2}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2} \Big|_{Q^2 = 0}.$$

#### **RMS** is the limit of the **form factor derivative** for $Q^2 \rightarrow 0$





#### G

#### High-Precision Determination of the Electric and Magnetic Form Factors of the Proton

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#### Mainz, A1 collaboration (1400 points)



 $\langle r_E^2 \rangle^{1/2} = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}$  fm,  $\langle r_M^2 \rangle^{1/2} = 0.777(13)_{\text{stat}}(9)_{\text{syst}}(5)_{\text{model}}(2)_{\text{group}}$  fm.

 $Q^2 > 0.004 \text{ GeV}^2$ 



G.I. Gakh, A. Dbeyssi, E.T-G, D. Marchand, V.V. Bytev, Phys.Part.Nucl.Lett. 10 (2013) 393, Phys.Rev. C84 (2011) 015212

²*→ 0*?



# Planned ep experiments



### Mainz ep elastic scattering



Cez

#### Mainz ep elastic scattering

$$\left\langle r_{E/M}^{2}\right\rangle = -\frac{6\hbar^{2}}{G_{E/M}\left(0\right)} \left.\frac{\mathrm{d}G_{E/M}\left(Q^{2}\right)}{\mathrm{d}Q^{2}}\right|_{Q^{2}=0}$$

1) Rosenbluth extraction

2) Direct extraction(assuming a function for FFs)

Spline  $\langle r_E^2 \rangle^{\frac{1}{2}} = 0.875(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}} \text{ fm},$  $\langle r_M^2 \rangle^{\frac{1}{2}} = 0.775(12)_{\text{stat.}}(9)_{\text{syst.}}(4)_{\text{model}} \text{ fm}$ 

#### Polynomial

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.883(5)_{\text{stat.}}(5)_{\text{syst.}}(3)_{\text{model}} \text{ fm},$$
  
 $\langle r_M^2 \rangle^{\frac{1}{2}} = 0.778(^{+14}_{-15})_{\text{stat.}}(10)_{\text{syst.}}(6)_{\text{model}} \text{ fm}.$ 













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#### The proton radius puzzle

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Abstract. By means of pulsed laser spectroscopy applied to muonic hydrogen  $(\mu^- p)$  we have measured the  $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$  transition frequency to be 49881.88(76) GHz [1]. By comparing this measurement with its theoretical prediction [2, 3, 4, 5, 6, 7] based on bound-state QED we have determined a proton radius value of  $r_{\rm p} = 0.84184(67)$  fm. This new value differs by 5.0 standard deviations from the CODATA value of 0.8768(69) fm [8], and 3 standard deviation from the e-p scattering results of 0.897(18) fm [9]. The observed discrepancy may arise from a computational mistake of the energy levels in  $\mu$ p or H, or a fundamental problem in bound-state QED, an unknown effect related to the proton or the muon, or an experimental error.





### Lamb shift and hyperfine splitting (1)



Rydberg constant



### Lamb shift and hyperfine splitting (1)



An e or  $\mu$  in S state has some probability to be inside the proton. The electric field (charge distribution) is modified by the proton size. The  $v_s$  and  $v_p$  transitions are affected by the proton size (few %)

## Lamb shift and hyperfine splitting

$$\Delta E_{\text{finite size}} = \frac{2\pi Z\alpha}{3} r_{\text{E}}^{2} |\Psi(0)|^{2} \qquad \text{Atomic wave function at the origin}$$

$$|\Psi(0)|^{2} \approx m_{\text{r}}^{3}, m_{\text{r}}(\mu p \text{ system}) \cong 186 m_{\text{e}}$$

$$\text{H radius : 60000 \times p \text{ radius}}$$

$$\mu \text{H Bohr radius is} \approx 200 \text{ times smaller: larger sensitivity!}$$

$$\frac{1}{4} hv_{\text{s}} + \frac{3}{4} hv_{\text{t}} = \Delta E_{\text{L}} + 8.8123(2) \text{meV}$$

$$hv_{\text{s}} - hv_{\text{t}} = \Delta E_{\text{HFS}} - 3.2480(2) \text{meV}$$

$$\Delta E_{\text{HFS}}^{\exp} = 22.8089(51) \text{ meV}$$

$$E_{\rm L}^{\rm th} = 206.0336(15) - 5.2275(10)r_{\rm E}^2 + \Delta E_{\rm TPE}$$

$$\Delta E_{\rm TPE} = 0.0332(20) \text{ meV}$$

$$r_{\rm E} = 0.84087(26)^{\rm exp}(29)^{\rm th}$$
 fm  
= 0.84087(39) fm



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Binding energy

# The Proton Size (Radius)



Cea