Cosmology

Pierre Astier

LPNHE / IN2P3 / CNRS , Universités Paris 6&7.

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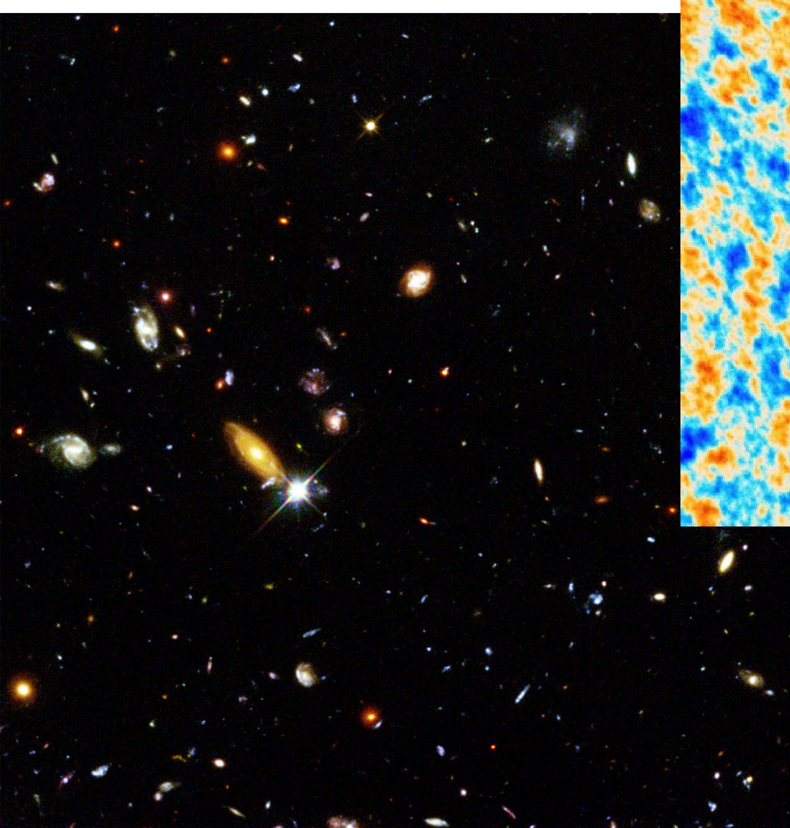

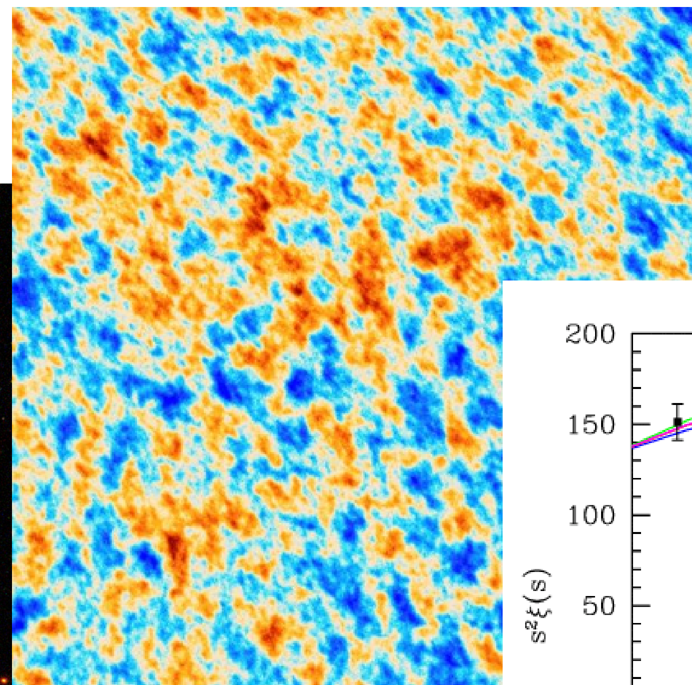

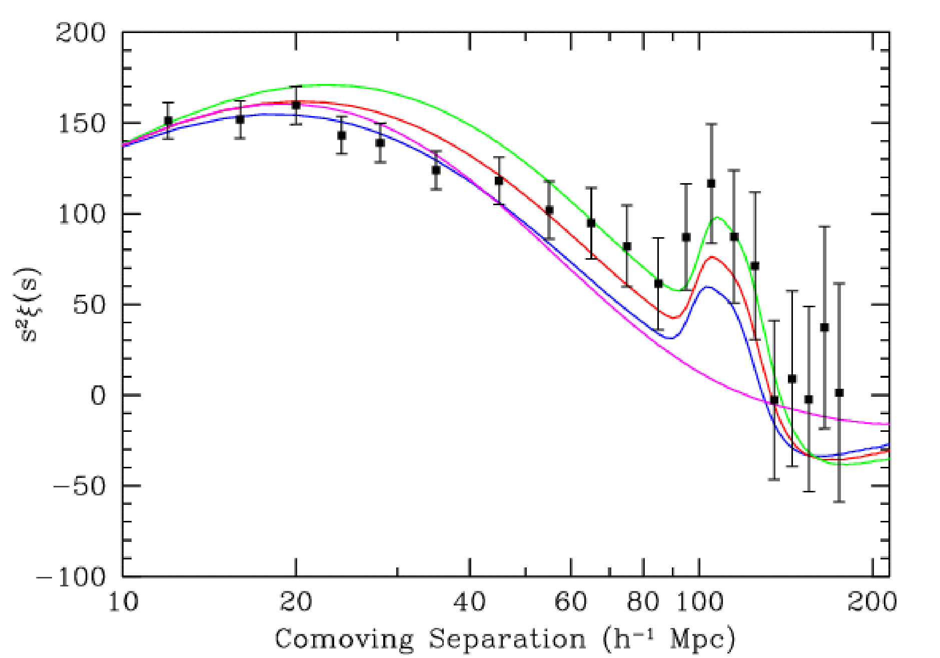

Two and a half lectures

- “Basics”
- The Planck mission and its results.
- Acceleration of expansion, with practical work

Textbooks :
- James Rich : “Fundamentals of Cosmology”
- John Peacock : “Cosmological physics”
- Scott Dodelson : “Modern Cosmology”
-

What is cosmology ?

- A branch of physics.
- That studies the universe as a whole:
  - History
  - Content, geometry (topology)
  - Formation of structures
  - Characteristic scales
  - ....

Only one universe:
one cannot replay
under varying
experimental conditions

- ~~experimental~~ observational science

  Messenger are (mostly) photons:
  - X
  - UV, visible, IR
  - deep IR , millimetric
  - radio, …

And gravitational wave astronomy is becoming real

Gravitation

On large scales, all other interactions vanish:

- Electro-magnetism : no forces, only waves
- Weak and strong forces have very short ranges
- However all interactions are at play in stars, galaxies, ….

Equivalence principle :

Gravitation couples to inertial mass

Gravitational and inertial forces are undistinguishable

Gravitation

On large scales, all other interactions vanish:

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Equivalence principle :

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Gravitational and inertial forces are undistinguishable

By the way, we have absolutely no understanding of the universality of free fall, which is probably the best established physical law, up to solar system scales.

Metric theory of gravitation

Trajectories in space-time only depend on initial conditions, not mass.

$\rightarrow$ one can encode gravitational forces into space-time geometry. Trajectories follow "shortest paths" i.e. geodesics.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Distance element

Metric tensor

$\rightarrow$ there are no special coordinate systems. All are equivalent.

Einstein equations

Function of $g_{\mu\nu}$

Cosmological constant

Energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma}_{\sigma} + \Lambda g_{\mu\nu} = 8\pi G \; T_{\mu\nu}$$

- This is General Relativity !
- Other metric theories are possible.
- Relates geometry (→ trajectories) to sources.
- 10 equations in general (4x4 symmetric)
- Covariant under general change of coordinates
- Non linear
- Radiation propagation is possible (and observed)

$$S = \int \left[\frac{R^{\sigma}_{\sigma}}{8\pi G} + \mathcal{L}_{\text{matter}} \right] \sqrt{-det(g_{\mu\nu})} d^4x$$

Invariant
Under a
Coordinate
Mapping change

Cosmological principle

The universe is homogeneous and isotropic

- no special position (Copernic) or direction
- … but no time invariance
- .. and spatial curvature is not defined

-> Friedman-Lemaitre-Robertson-Walker metric:

$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(\sin^2\theta d\theta^2 + d\phi^2)\right)}$$

Scale factor

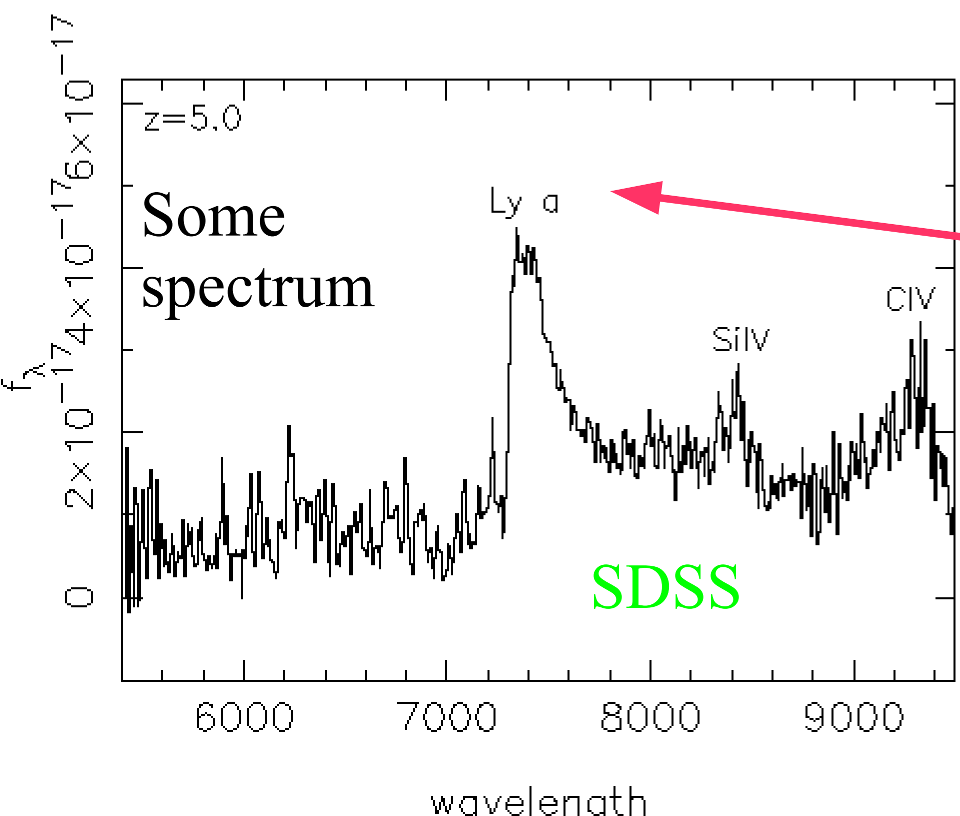
Comoving coordinate

k = -1,0,1 (curvature sign)

Redshift z

$$1 + z \equiv \frac{\lambda_{reception}}{\lambda_{emission}} = \frac{a(now)}{a(emission)}$$

Assumes that emitter and receiver are both comoving (i.e. "attached" to matter)



Redshift allows us to measure scale factors !

Ly α : 1216 Ang. In the lab
z = 7400/1216 - 1 =~ 5.0

Shift to the red:
- a(t) increases with t
- expansion !

And also: time dilation ...

Friedman equation(s)

GR: Einstein Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma}_{\sigma} + \Lambda g_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

FLRW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(\sin^2\theta d\theta^2 + d\phi^2)\right)$$

$$\boxed{H^2(t) \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}}$$

In principle sufficient, once specified how density (ρ) depends on a(t).
Alternatively :

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

A negative pressure can accelerate expansion.

Historical
& Newtonian
parenthesis

(

Our cosmological model : founding stones

1915 : Albert Einstein proposes General Relativity
1922 : Alexander Friedman proposes evolving universe models
1927 : Georges Lemaître proposes evidence for expansion
1929 : Edwin Hubble : "the faster, the fainter"

Recession velocity
of nebulae
vs "distance"

velocity

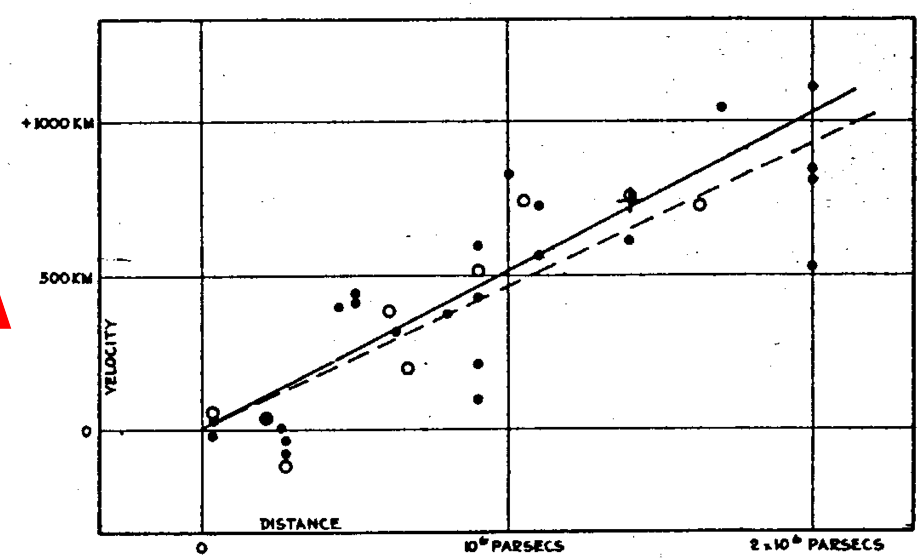
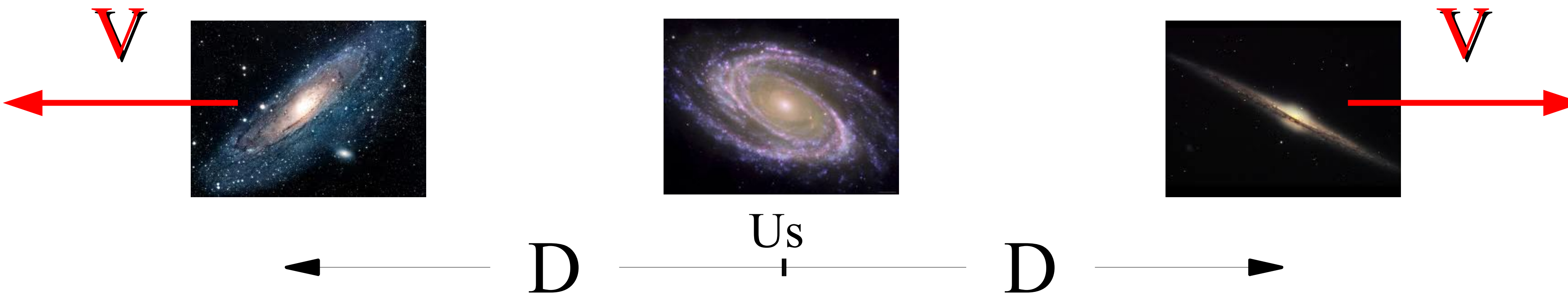


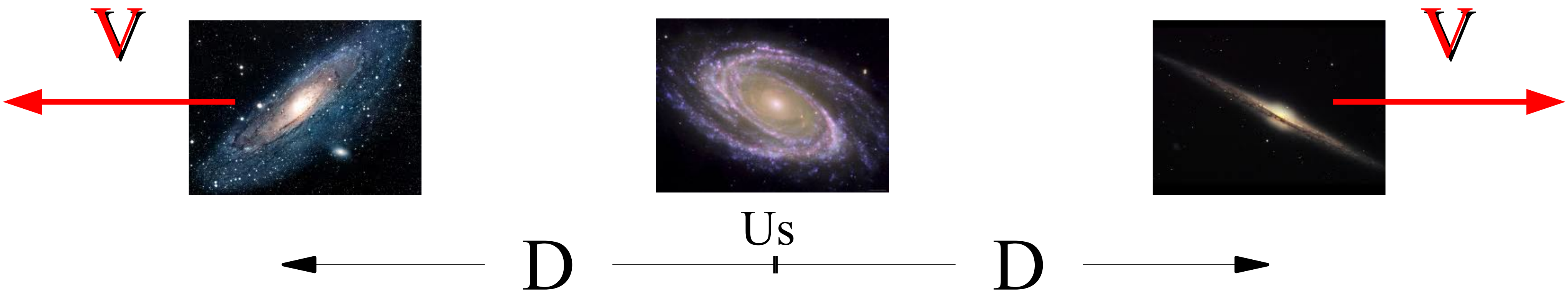
FIGURE 1

Distance (from flux)

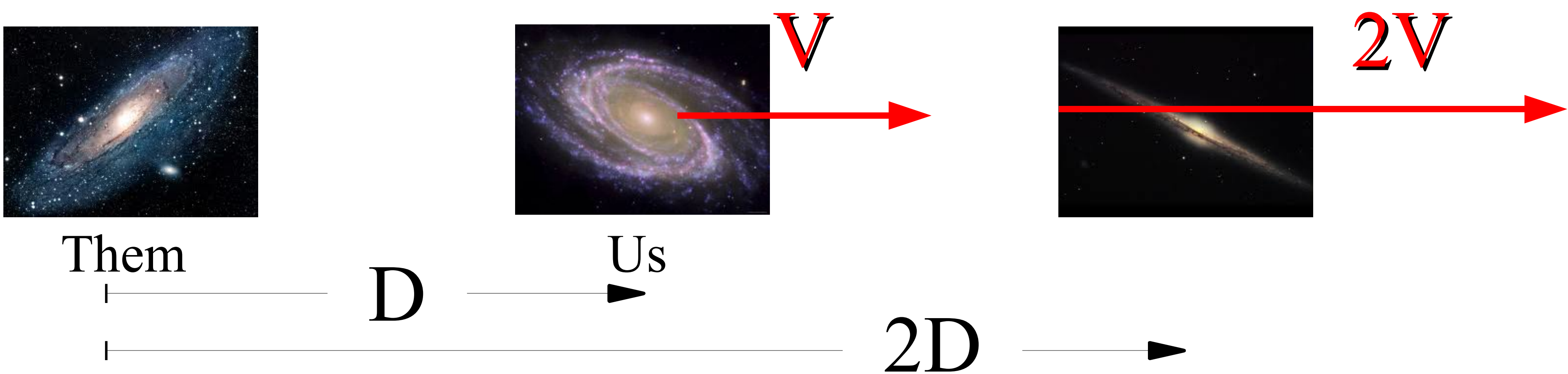
Expansion



Expansion



Let us change our view point



Expansion

Cosmological principle:

No favoured direction nor location

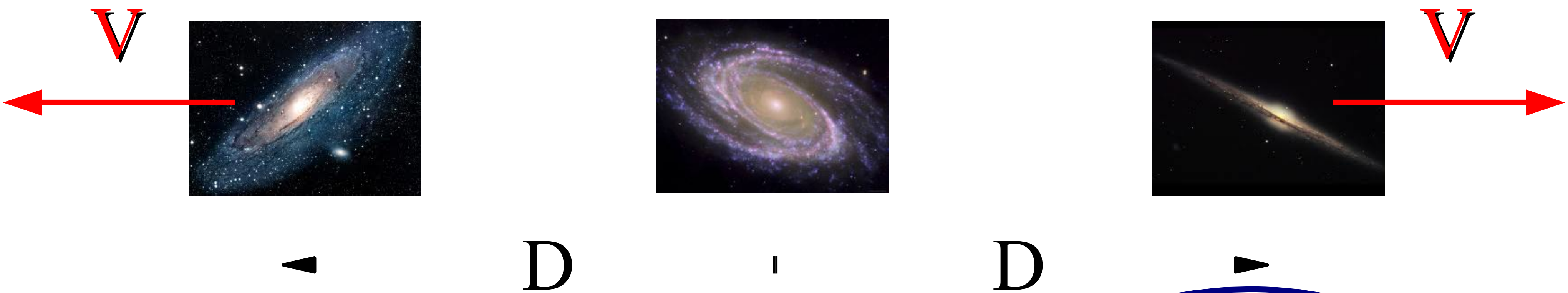
⇓

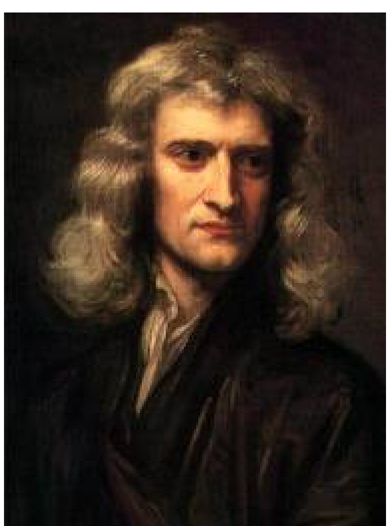
Velocity and distance are proportional

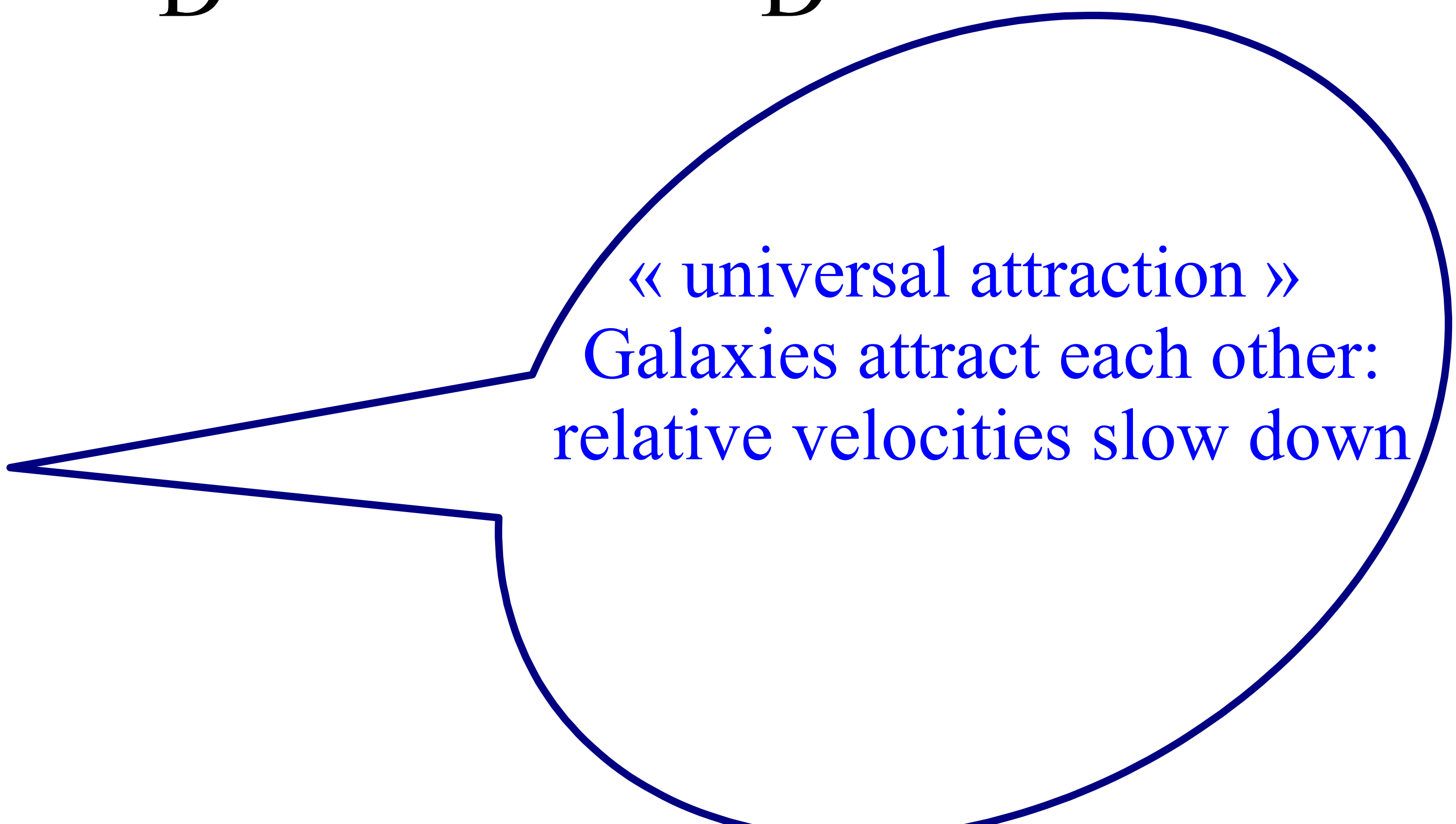
(to first order)

No expansion would be a particular case

Expansion : deceleration ?

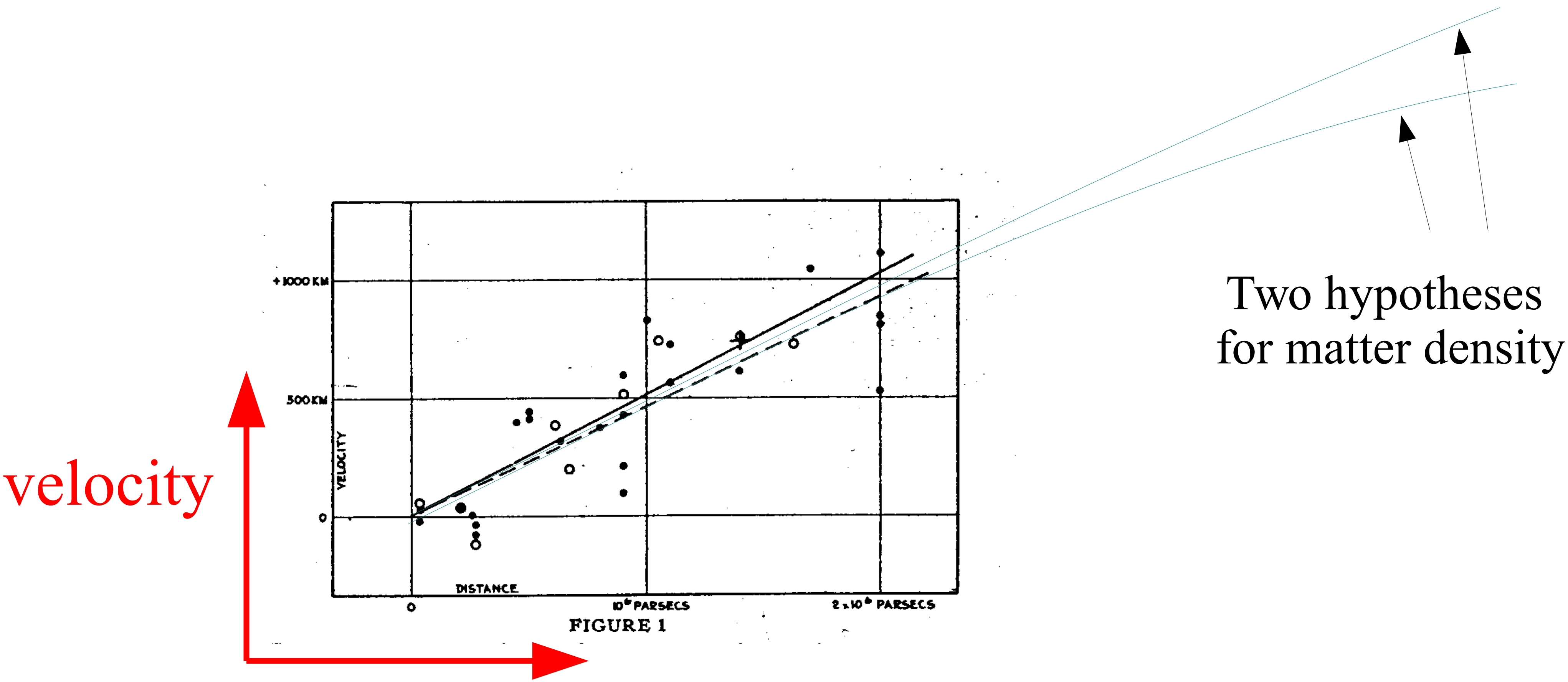






So

- $V = H\,d$ is a signature of the expansion of the universe
- The deceleration of expansion with time (or distance) encodes matter (or more generally energy) density.



Distance (from flux)

Historical & Newtonian parenthesis

)

The fate of expansion ? It depends ...

$$\boxed{H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}}$$

"initial" conditions = present Conditions

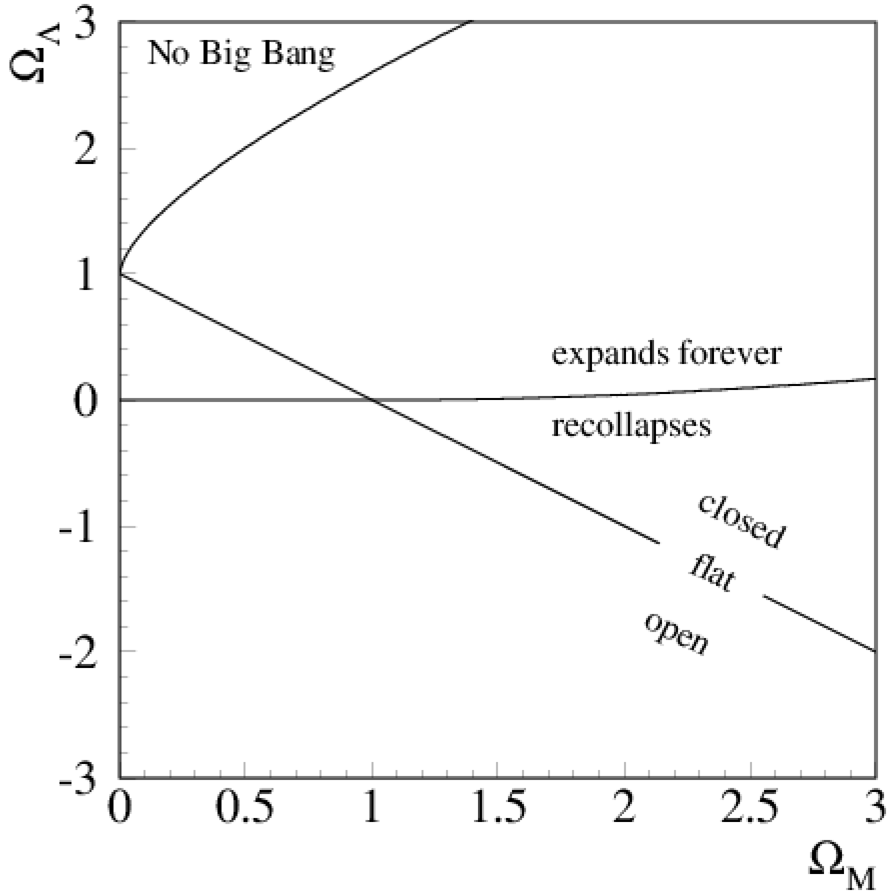
$$H_0 = \left(\frac{\dot{R}}{R}\right)_0$$

$$\Omega_M = \frac{8\pi G}{3H_0^2} \rho_{M,0}$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_k = -\frac{k}{R_0^2 H_0^2}$$

$$\Omega_M + \Omega_\Lambda = 1 - \Omega_k$$



Densities in cosmology

Density means "energy density" (i.e. mass + kinetic energy)

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Critical density: the one which makes the universe flat, i.e. k=0.

Dimensionless density (today): $$\Omega_X \equiv \frac{\rho_X}{\rho_{\rm crit}} = \frac{8\pi G \rho_X}{3H_0^2}$$

"Physical" density: $$\Omega_X h^2 = \frac{8\pi G \rho_X}{3H_{\rm ref}^2}$$

Common convention : $$h \equiv \frac{H_0}{100\ \mathrm{km/s/Mpc}}$$

The “equation of state”

$$\boxed{H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}}$$

To integrate this, you have to specify how ρ depends on a, or t .

$$d(\rho V) = -p dV = \rho dV + V d\rho$$

Definition of pressure

$$\Rightarrow \quad \dot{\rho} = -3H\rho(1+w)$$

w constant

$$\rho = \rho_0 a^{-3(1+w)}$$

Equation of state: $w \equiv \frac{p}{\rho}$

Differential equations for expansion

Friedman equation $$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

Acceleration equation $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

Energy conservation equation $$\dot{\rho} = -3H(\rho + p)$$

Exercise : show that these 3 equations are redundant

Simple solutions

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \cancel{\frac{\Lambda}{3}} - \cancel{\frac{k}{a^2}}$$

Set k=0 (flat universe), Λ=0 (could be integrated into ρ)

ρ scales as $a^{-3(1+w)}$. w (assumed constant) is called “equation of state”

Radiation	Matter	Λ
w=1/3	w=0	w=-1
$\rho \propto a^{-4}$	$\rho \propto a^{-3}$	$\rho = C^{st}$
$a \propto t^{1/2}$	$a \propto t^{2/3}$	$a \propto \exp(t/\Lambda^{1/2})$

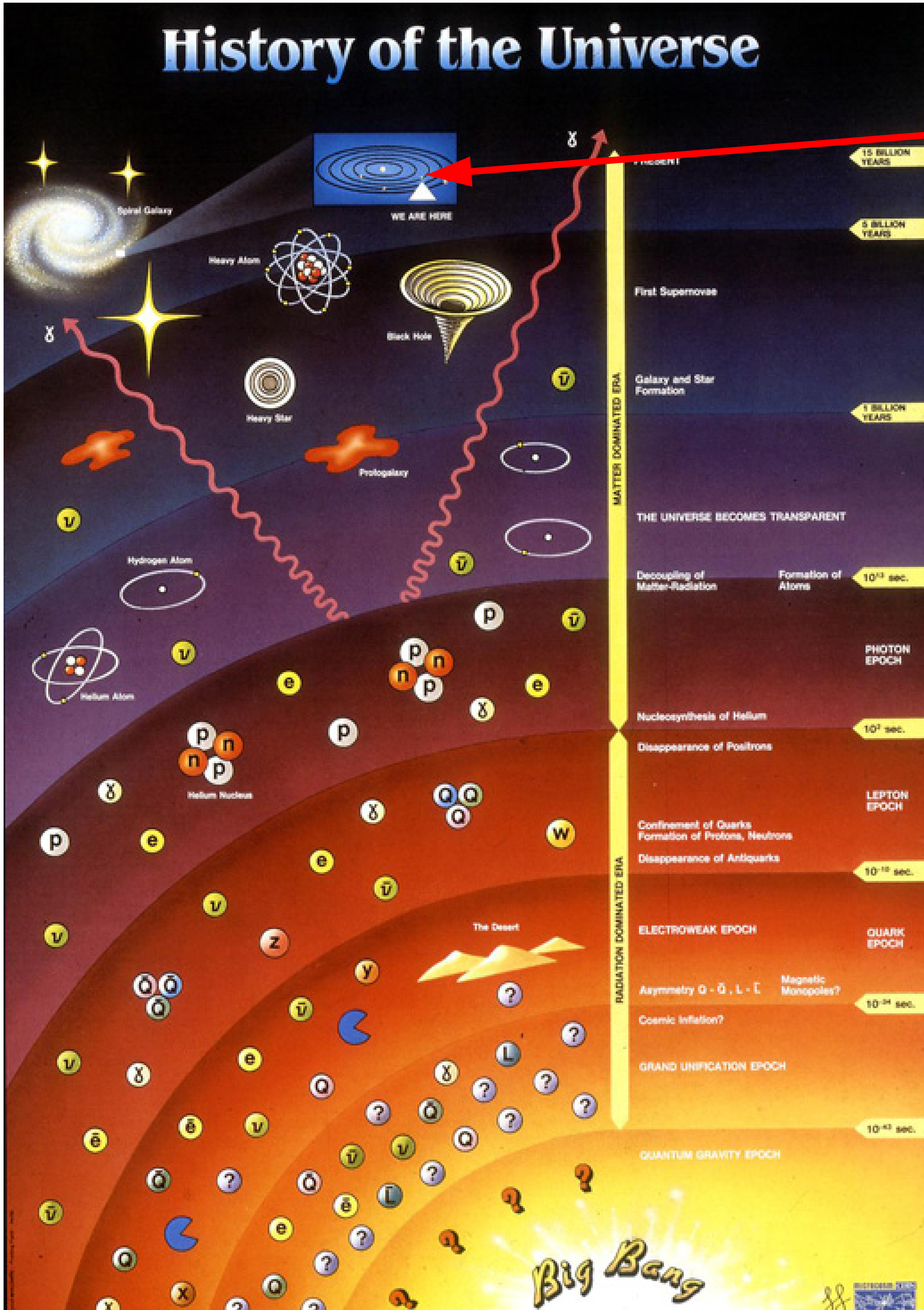
If the universe expands there could an "initial singularity"

Pointed out by Lemaître (1927)

This is true for almost any "reasonable" content today.

This initial singularity is commonly called the Big Bang.

It violates time invariance: global energy conservation
is not realized.



You are here

Structures form all the way

400 000 y

Atoms form

3 mn

Light nuclei form

Few s

Positrons disappear

….

The Big Bang sketch

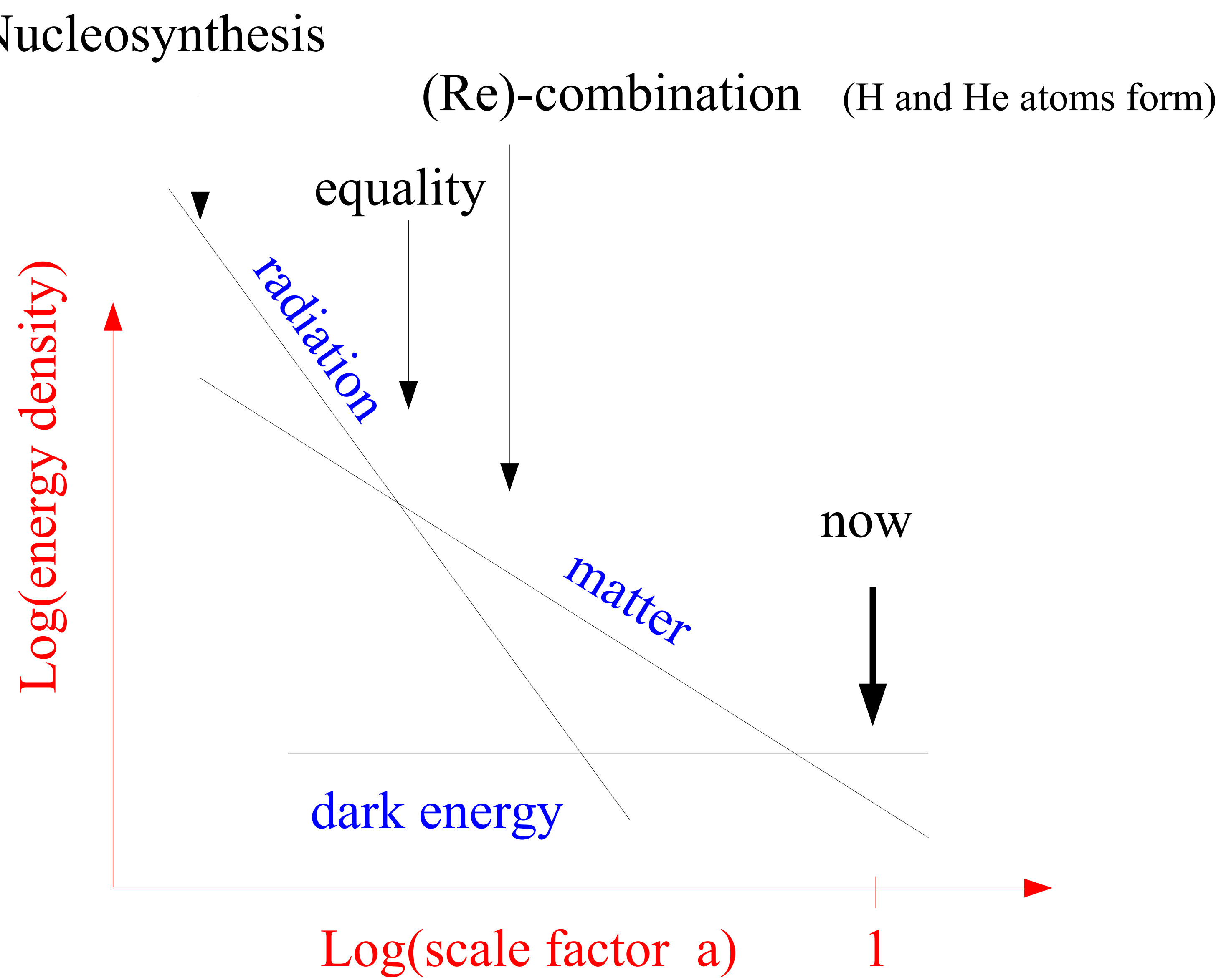
Qualitative cosmology

As time goes:

- Temperature decreases
- Density decreases
- Lighter and lighter particles “freeze out” (cst comoving density)
- Bound states form (with smaller and smaller binding energies)
- Contrast to homogeneity increases

This is why high-energy accelerators are related to Big Bang

A brief history of the universe



How long since Big Bang ?

$$H_0 \equiv \left.\frac{\dot{a}}{a}\right|_{\rm today}$$

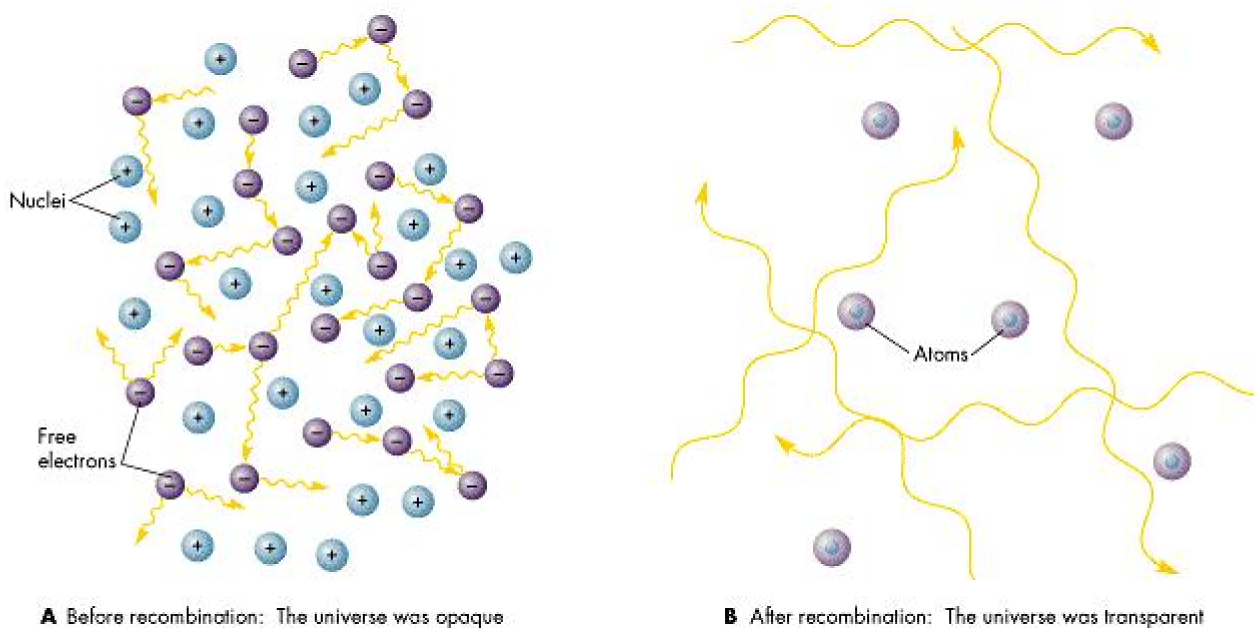
$$t_{\rm now} - t_{\rm BB} = O(1)/H_0$$

$$H_0^{-1} = 13.95\ 10^9 y\ \left(\frac{H_0}{70\ {\rm km/s/Mpc}}\right)^{-1}$$

Observational evidence of the hot Big Bang scenario

- The Cosmological Microwave Background.
- The cosmological abundance of light elements.
- The evolution of large scale structures.
- The age of oldest stars.

The CMB emission
(cosmic microwave background)



Before recombination After recombination

“Recombination” should be just called “combination”
… but is always called recombination.

CMB is a fossil remain of the hot big bang.

When did it happen

Order 0: when energy of photons ~ 13.6 eV (H binding energy)

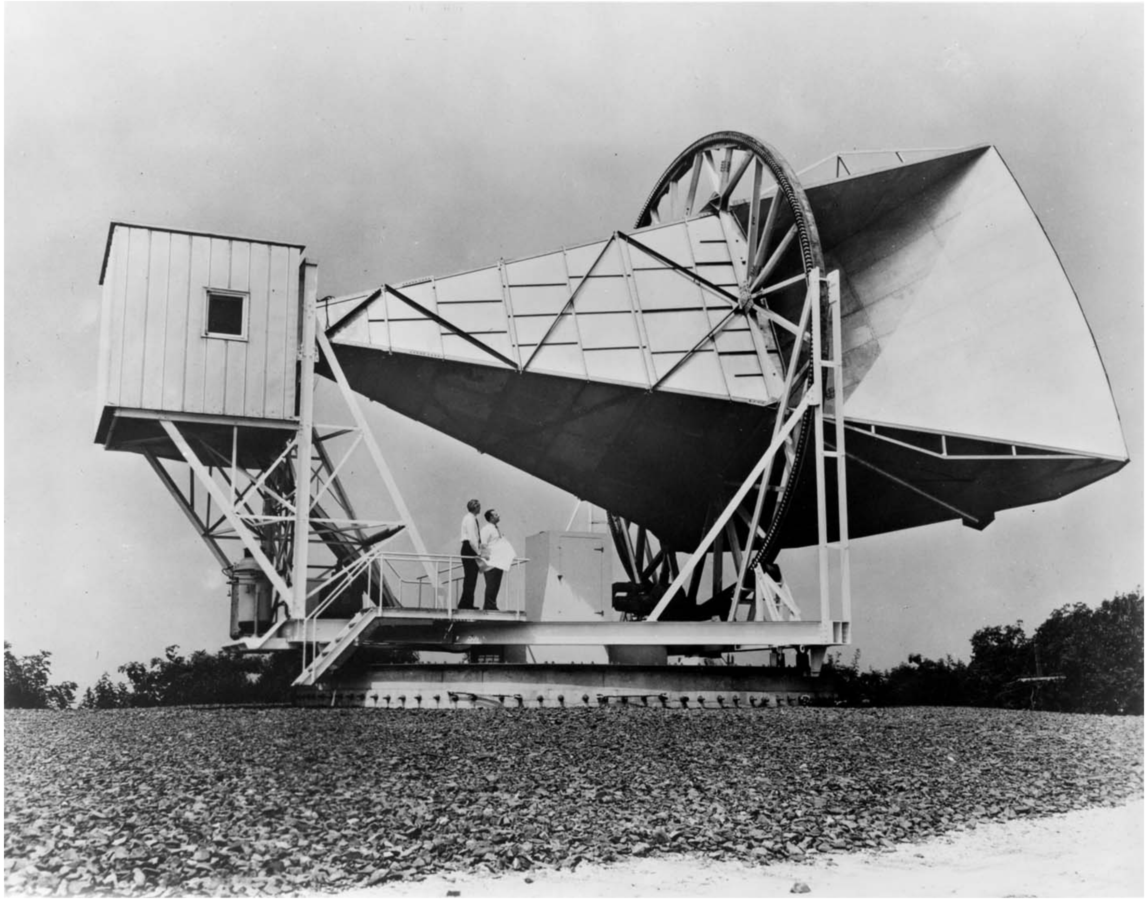
Order 1 : when there were as many photons >13.6 eV than electrons and protons

$$T_{rad} = 13.6\ eV\ \log\left(\frac{N_b}{N_\gamma}\right) \simeq 0.7\ eV \simeq 7000K$$

Order 1.5 : replace 13.6 by 3/4*13.6 (n=1 → n=2). Find 5000 K

Beyond : involved atomic physics and numerical codes. Find 3000 K
→ emitted ~ 380,000 years after BB

CMB detection (and identification) already 53 years !

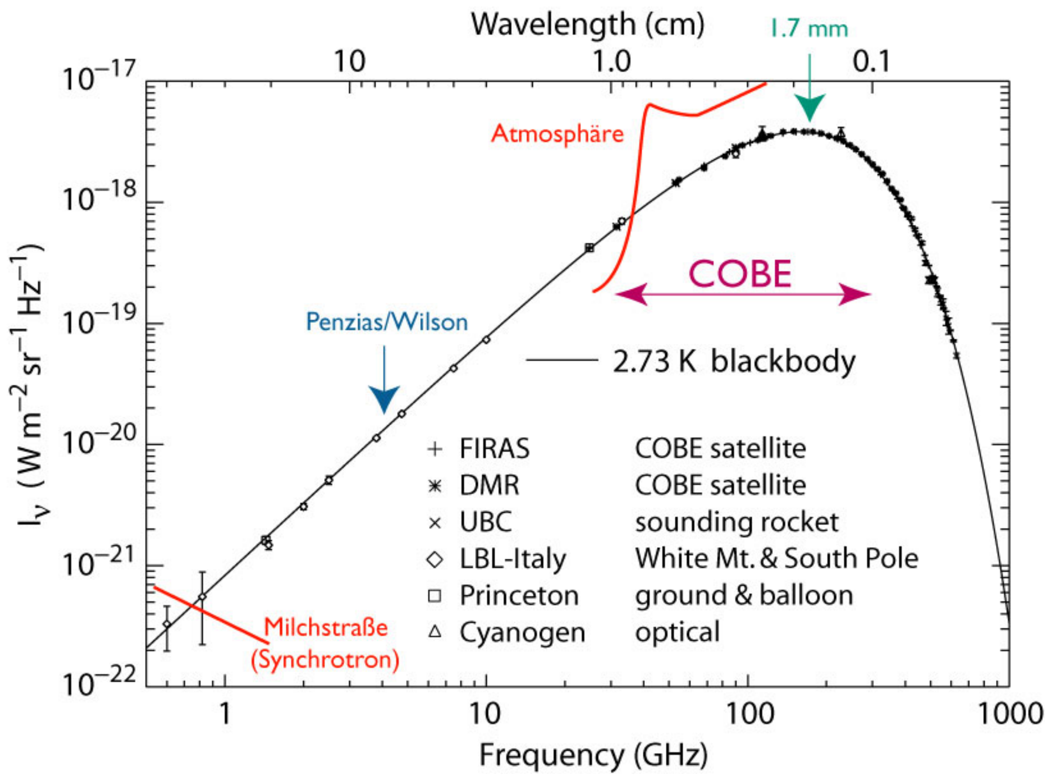


(Penzias & Wilson, Bell Labs, 1965)

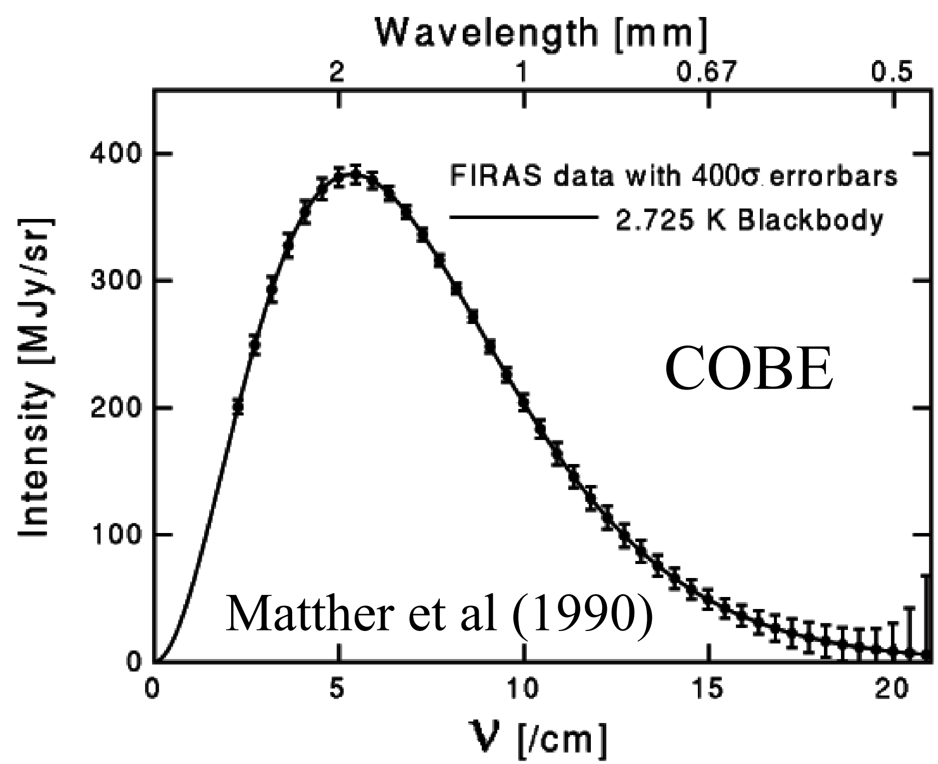
A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and

CMB spectrum: it has to be thermal...
...and it is thermal



(Prof. Dr. Karl - Heinz Kampert, Uni Wuppertal)



T = 2.726 +/- 0.005 (sys dominated)

the most precise cosmological measurement still

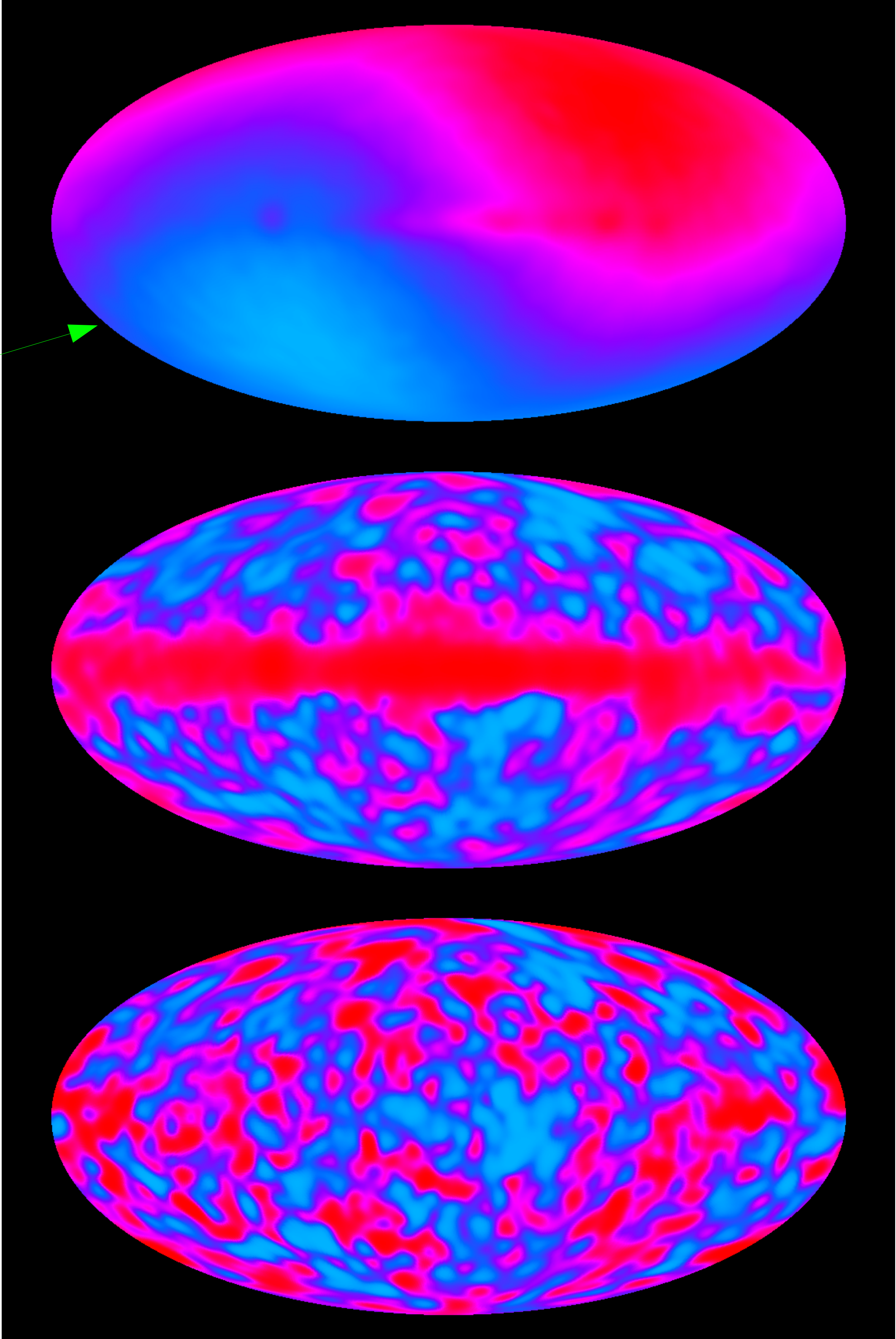
→ also delivers photon density : 413 cm^{-3} today

CMB anisotropies

Dipole due to our peculiar velocity

Milky Way emission

$$\frac{\delta T}{T} \sim 10^{-5}$$

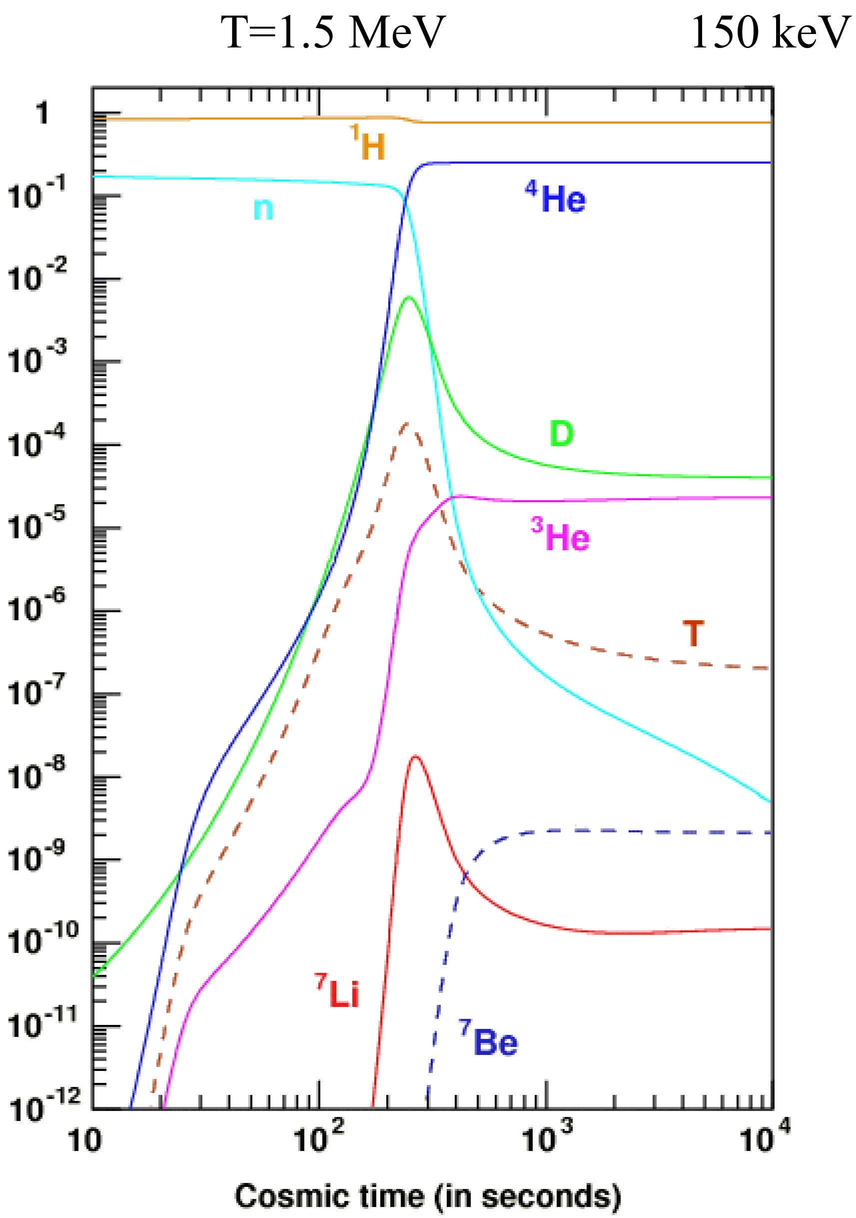


(COBE DMR, Smoot et al, 1992)

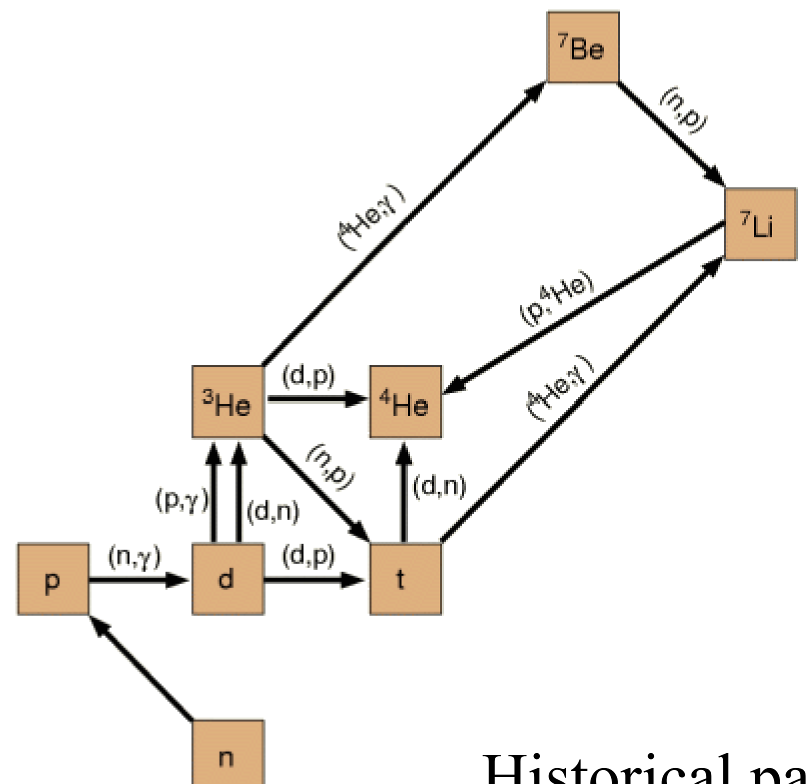
Observational evidence of the Big Bang scenario

- The Cosmological Microwave Background.
- The cosmological abundance of light elements.
- The evolution of large scale structures.
  We'll come to that soon
- The age of oldest stars:
  The age of stars can be evaluated using stellar evolution models
  The oldest observed stars are ~13 Gyr old.

Big Bang Nucleosynthesis



Nucleosynthesis i.e. forming light nuclei
-starts at T ~ MeV (t~100 s)
- stops when density gets too low
(or run out of neutrons)



Historical paper:
Alpher, Bethe & Gamow (1948)

Nucleosynthesis (2)

Main drivers :

$\eta = N_{baryons}/N_{photons}$

Expansion rate (depends on the number of neutrino flavours)

Measurements of abundances :

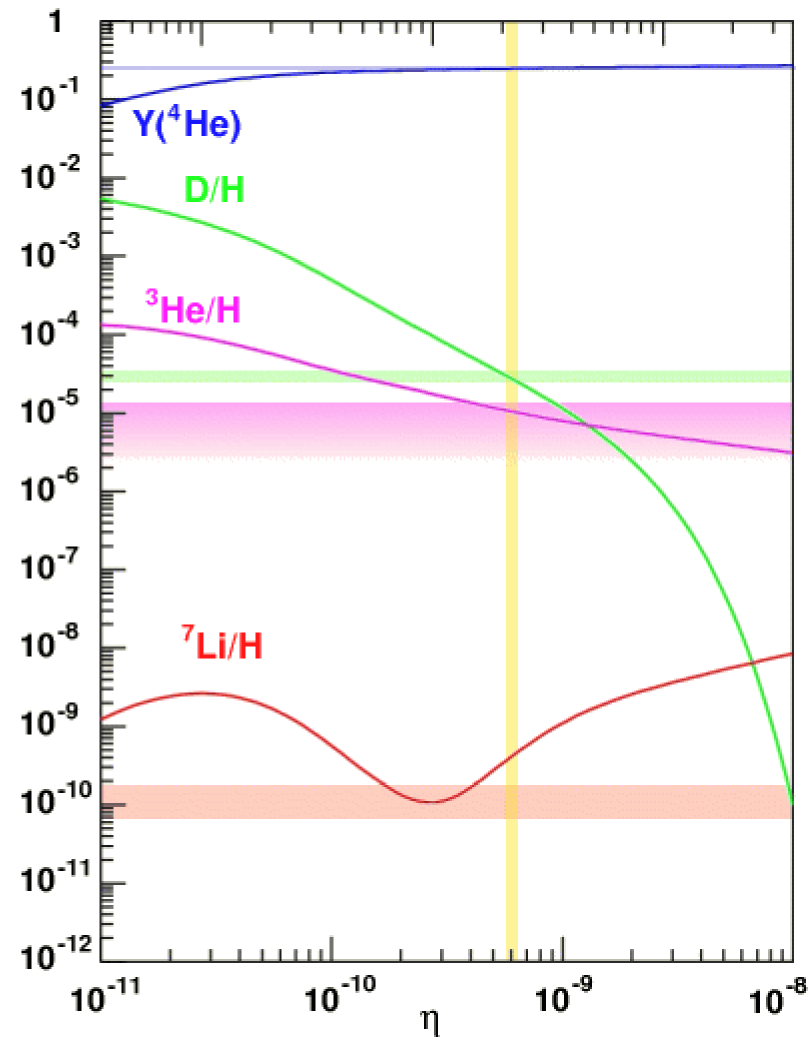
Helium fraction is 0.24 (safe)

D/H is hard to measure (settled now).

Li is destroyed in stars.

Bottom line:

- $\eta = 6\ 10^{-10}$ explains measured abundances
- Photon density is known
  → yields baryon density



E. Vangioni (IAP)

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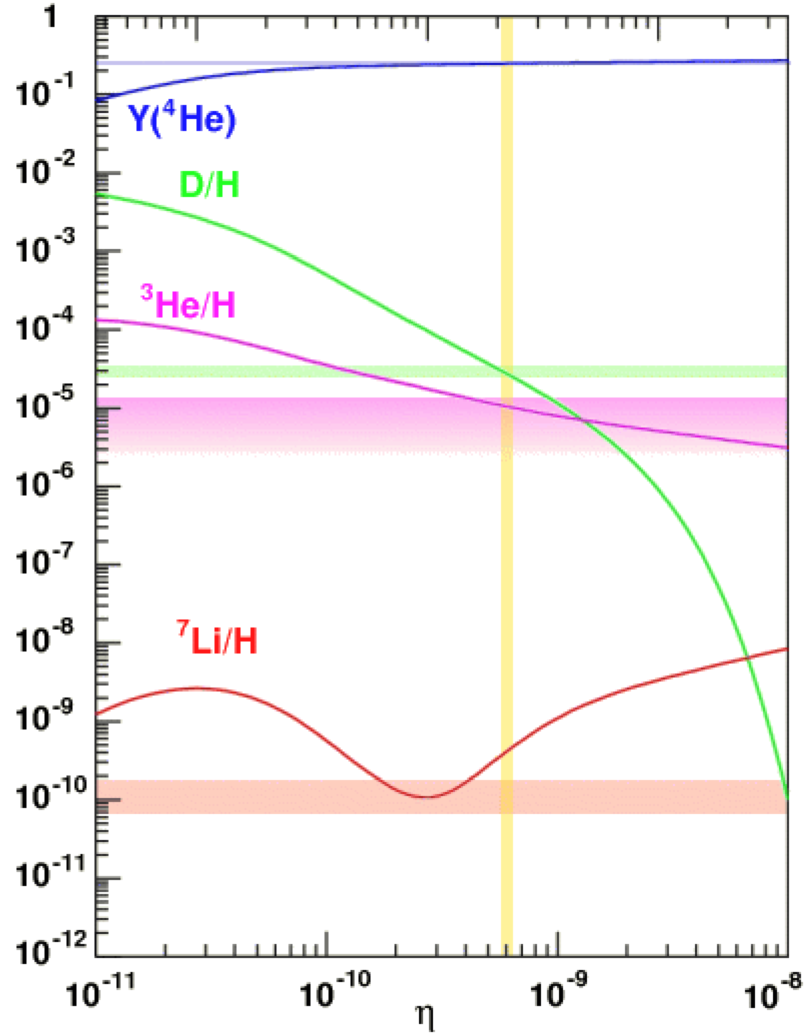
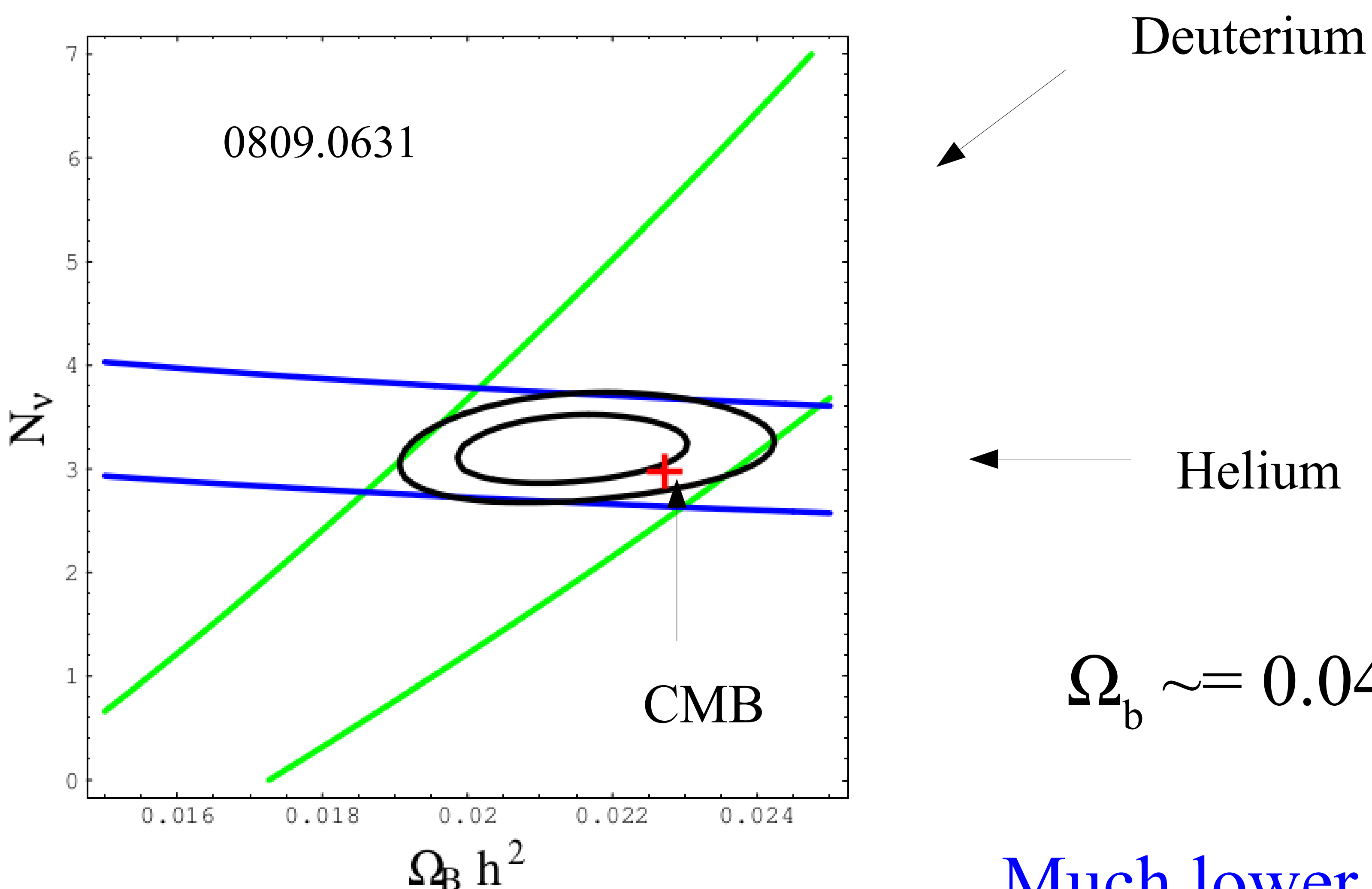


We basically do not understand this number

E. Vangioni (IAP)

Nucleosynthesis (3)



$\Omega_b \sim= 0.045$

Much lower than Ω_m

Observational evidence of the Big Bang scenario

- The Cosmological Microwave Background.
- The cosmological abundance of light elements.
- The evolution of large scale structures.
  We'll briefly discuss that
- The age of oldest stars:
  The age of stars can be evaluated using stellar evolution models
  The oldest observed stars are ~13 Gyr old.

So, there was a hot Big Bang about 13 Gyr ago

Or, …. everything looks like there was one ….

Two relics are well explained by a hot Big Bang:

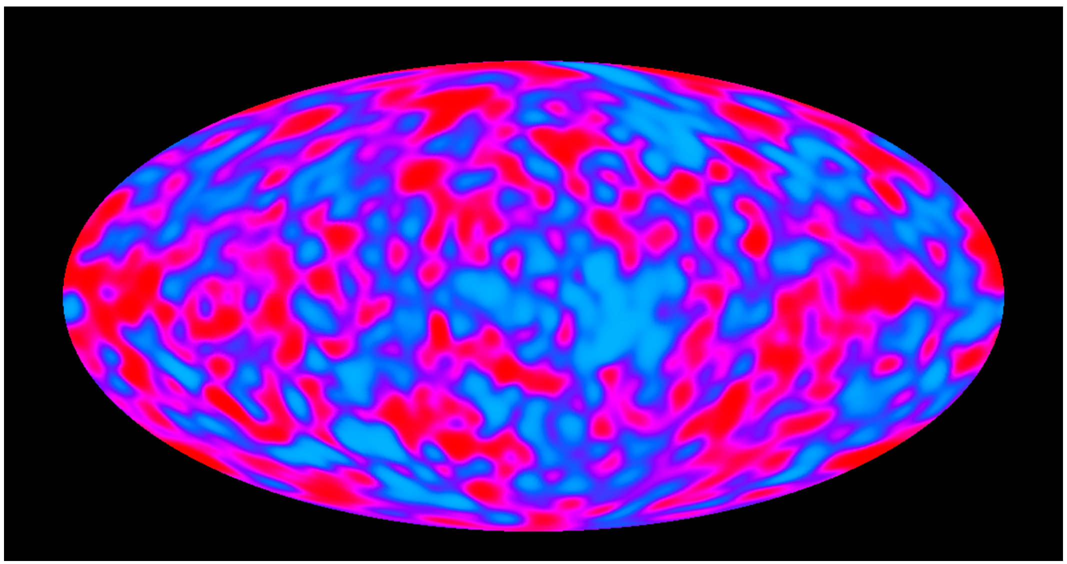
- Light nuclei (~3 mn)
- CMB (~ 400 000 yr), thermal and isotropic.

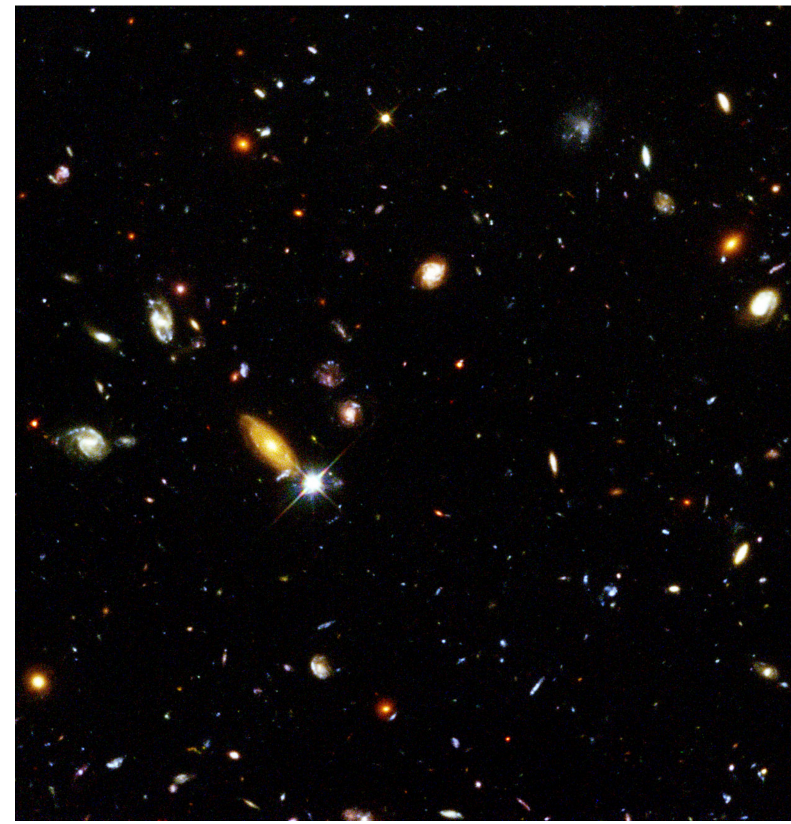
~~Experimental~~ Observational Program :

- figure out the (average) content
- understand the formation of structures.

Formation of structures

Practical question: how are these 2 picture related (quantitatively)





Formation of structures is the result of competition between attraction by gravity and pressure and expansion.

Homogeneity : Friedman equation(s)

GR: Einstein Equations

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\sigma_\sigma + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$$

FLRW metric

$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(\sin^2\theta d\theta^2 + d\phi^2)\right)}$$

$$\boxed{H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}}$$

This is General Relativity for an homogeneous and isotropic universe

a(t) : scale factor. By convention a(now) = 1.
ρ is the (energy) density. One could integrate Λ in it.
k = -1,0,1 is the sign of curvature

Beyond homogeneity : Perturbations

Perturbations describe fluctuations beyond the homogeneity:
- density perturbations
- metric perturbations (expressed with gravitational potentials)
- coupling between the two (Einstein equation)

We know from CMB observations that early perturbations are small

→ First order perturbations will capture the physics (before recombination)
→ Linear differential equations. Spatial coordinates are often handled in Fourier space.
→ Independent Fourier modes.

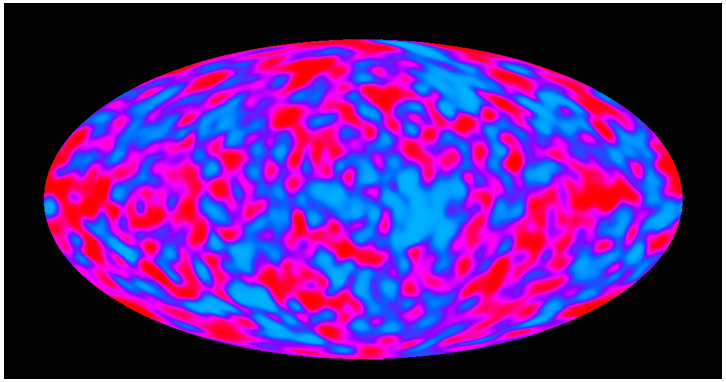
Density perturbations

Definition: $$\delta(r) = \frac{\rho(r)}{\langle\rho\rangle} - 1$$

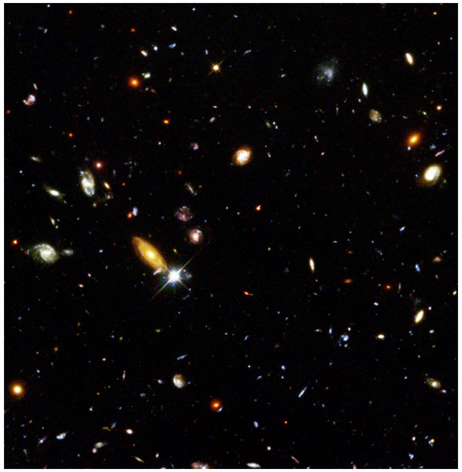
- Physics at play:
  - Gravitation (positive perturbations tend to grow)
  - Expansion (!)
  - Pressure (from photons on charged particles)
  - Sound waves in the primordial plasma
  - Transport (by photons and neutrinos)
  - length limits imposed by causality
  - ….

Density perturbations

- In almost all conditions, density perturbations do grow : δ(ρ)/ρ grows, as density decays.
- So the history of the universe is not only a decrease of average density, it is also an increase of constrast



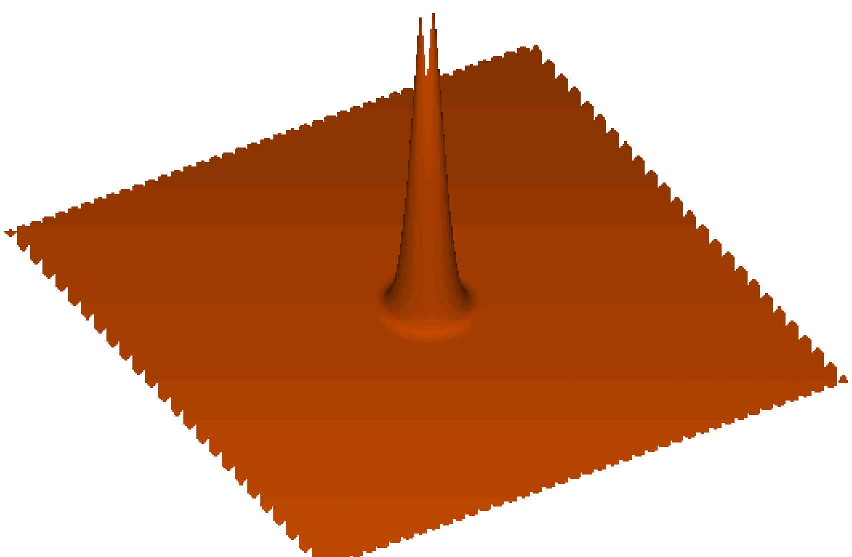
$\delta T/T \sim 10^{-5}$, z~1100



$\delta\rho/\rho \sim 1$, z~1

Relics of early perturbations: sound horizon

- Sound waves in the early plasma (before recombination)
  - The dominant contribution to the CMB anisotropies on small scales (< ~2 degrees).
  - Leaves forever a preferred length in the matter density fluctuations



Sound waves propagate in the primordial plasma until recombination where the pattern just freezes.

Sound horizon: distance travelled by sound waves Until recombination

Relics of early perturbations: the horizon at equality

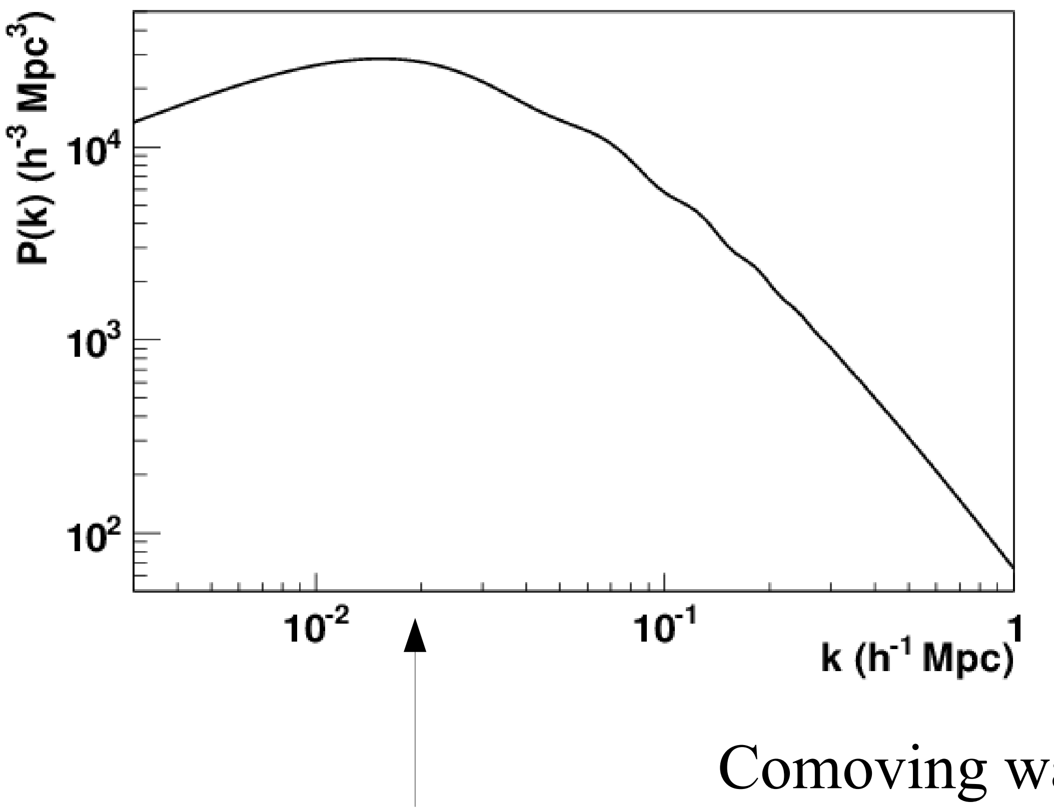
- Perturbations can be washed out by radiation...
- ...only if they are smaller than the horizon
- … and if the expansion is driven by radiation.
- Two limiting regimes:
  - Radiation dominated (early) vs matter-dominated (later) expansion
  - Smaller or larger than the (evolving) horizon.

Relics of early perturbations: the horizon at equality

Power spectrum of matter:

$\delta(k) = FT(\delta(r))$

$P(k) = |\delta(k)|^2$



Equality:
when matter and radiation densities are equal

Comoving wave number

(comoving) horizon size at equality

Why measuring that is important ?

- This horizon size is a function of matter and radiation density.
- Do we know the radiation density ?
- How well ?

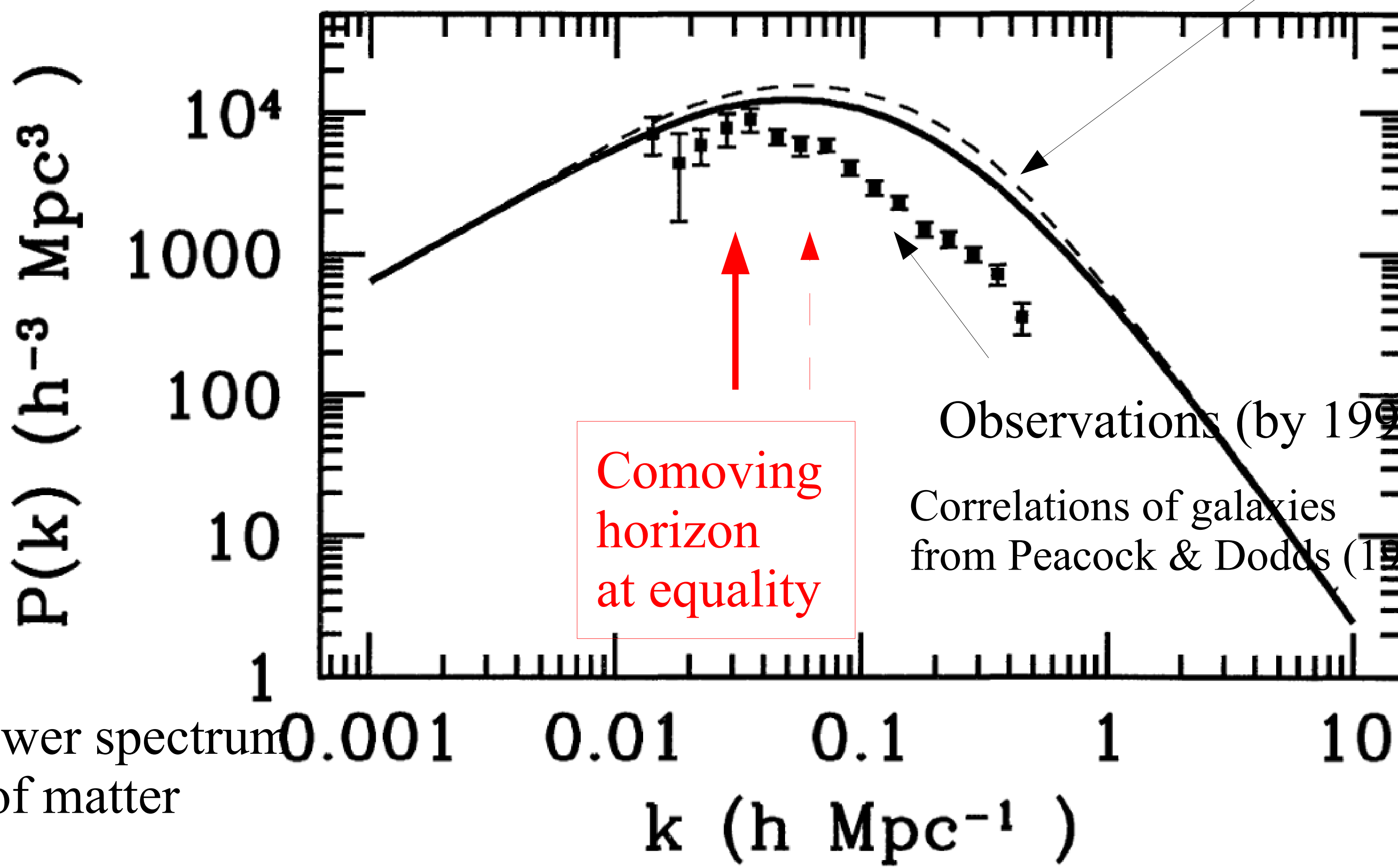
Why measuring that is important ?

- This horizon size is a function of matter and radiation density.
- Do we know the radiation density ?
- How well ? From CMB temperature !
- So if we know Ω_m/Ω_{rad}, we deduce Ω_m.

Matters correlations nowadays
Model vs observations

Prediction for $\Omega_M = 1$

$\Omega_M = 0.3$



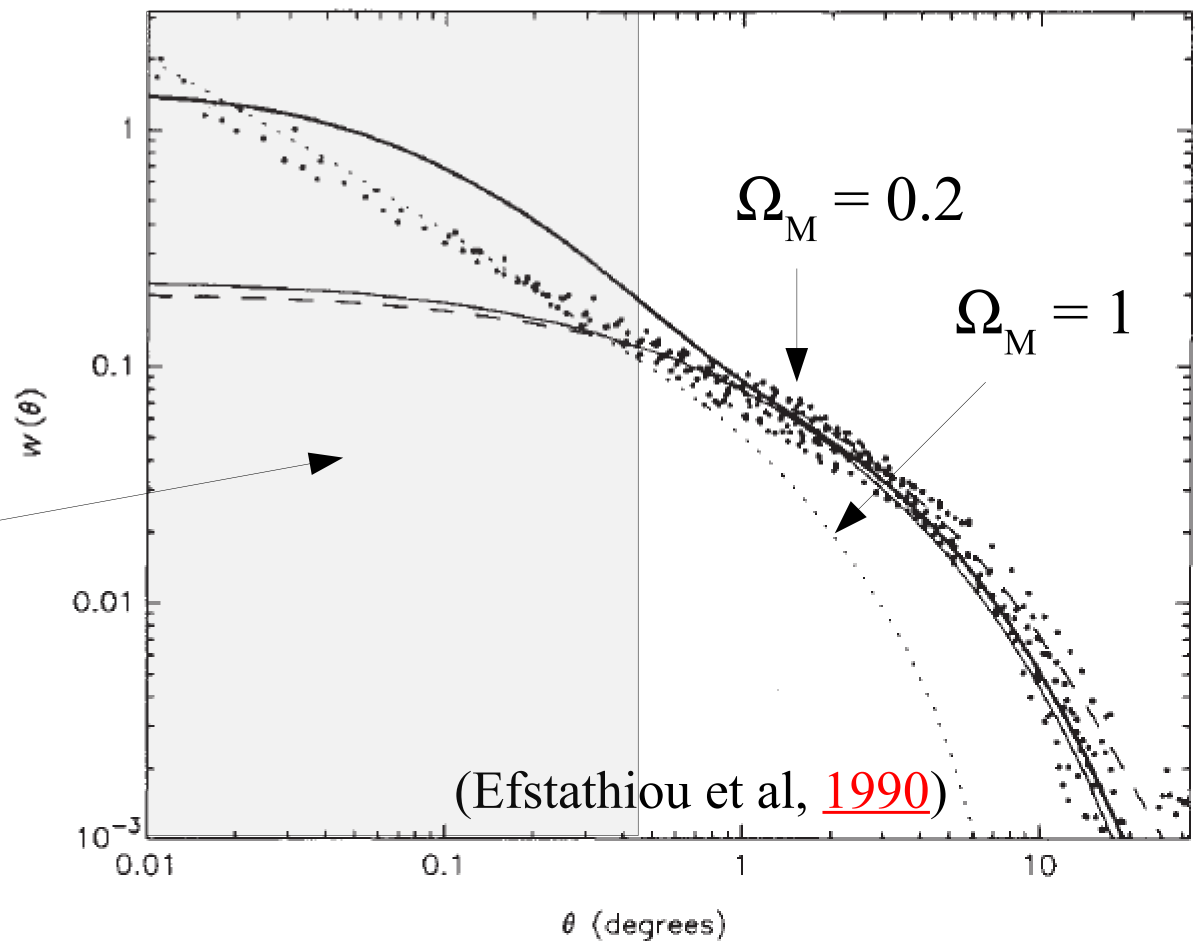
Power spectrum of matter

Figure from “Modern Cosmology” (Dodelson, Academic Press)

Matters correlations in the nearby universe : model vs observations

Angular correlation function of galaxies

Scales are too small for simple predictions to be reliable



The correlations of galaxies have challenged $\Omega_M = 1$ for more than 20 years !

One dark matter indication

Baryonic matter has density $\mathbf{\Omega_b}$ ~0.05

primordial nucleosynthesis, Helium fraction.....

Matter has density Ω_M ~ 0.3

from matter density perturbation correlations

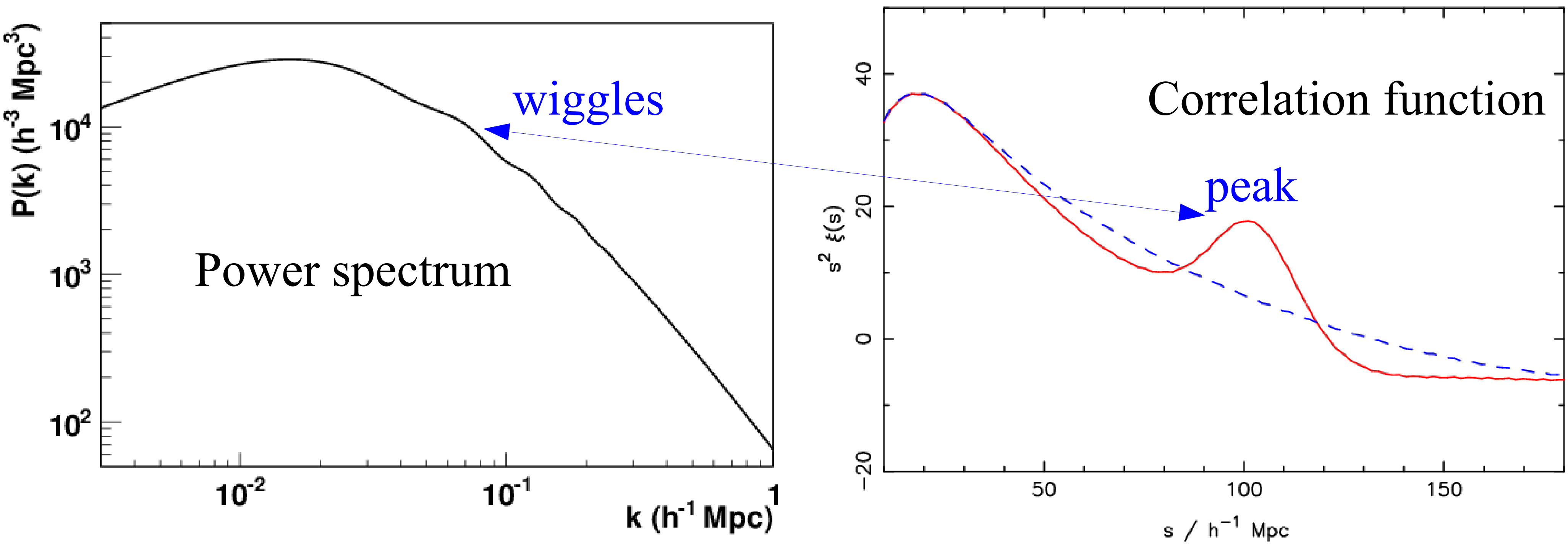
~~Some~~ most matter is non-baryonic

Summary

- There is ample evidence in favour of the hot big bang scenario.
- Big Bang Nucleosynthesis indicates that the baryon density is ~ 5% of the critical density
- There are two physical lengths which are relics from the growth of early perturbations
  - The sound horizon at recombination
  - The (event) horizon at matter-radiation equality
- The matter density is ~ 0.3 of the critical density.

More slides

Correlation function and power spectrum



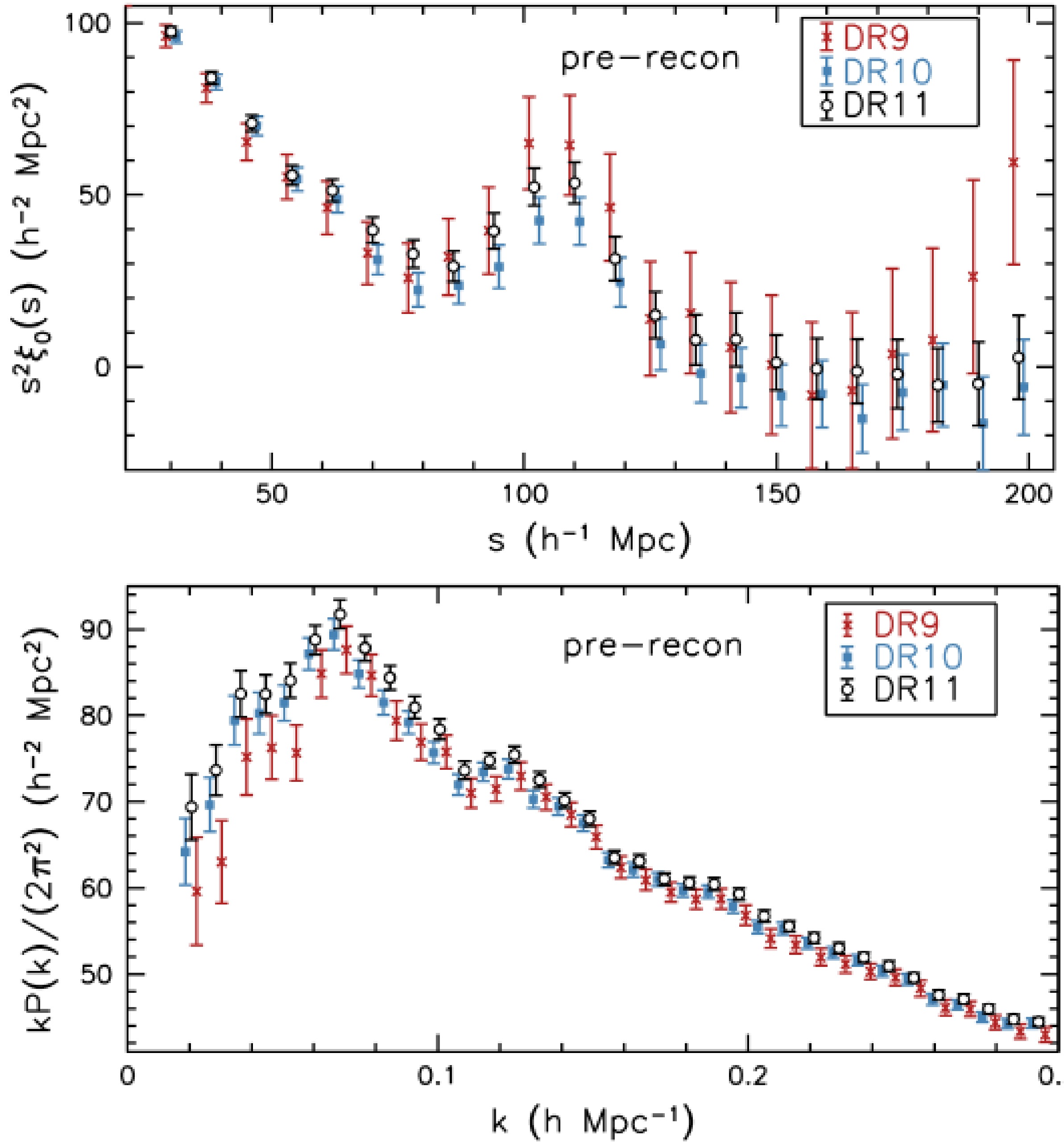
A single peak in the correlation function
→ harmonic peaks in the power spectrum

Measurements using Galaxies as “tracers” of the matter field

BOSS galaxy
Redshift survey

1312.4877

Successive data releases (DR) correspond to Increasing amounts of data.



Initial power spectrum

Definition of P(k) $< \delta\rho(\mathbf{k})\delta\rho(\mathbf{k}') >= (2\pi)^3 P(k)\delta(\mathbf{k}-\mathbf{k}')$

No natural scale → has to be a power law

$$P(k) = Ak^n$$

n=1 is called Harrison-Zeldovich-Peebles spectrum.
("scale invariant" because $k^3P_\phi(k)$ (dimensionless)
does not depend of k)

→ We expect n=~1. It turns out that we measure n ~= 0.96
We are fairly sure we understand the small difference.

Evolution of perturbations

Computations to first order. Few results to second order.
Complex subject: typically more than 50 pages in cosmology textbooks.

Only a qualitative discussion here...

Simple example: evolution of matter perturbations in a matter dominated universe without radiation:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_M\delta$$

The evolution of a perturbation is (in this case) independent of its size
Two solutions: one decaying (uninteresting), one growing $\delta \propto a(t)$
This is what happens after recombination.

Various perturbation growth regimes

Two limiting regimes:

- Causal vs not causal (wavelengths larger or smaller than the horizon)
  On small scales, pressure effects tend to oppose gravitational collapse.

- Radiation dominated vs matter dominated

So, the horizon size at matter-radiation equality is imprinted on the matter fluctuations.

Horizon size at matter-radiation equality

Comoving horizon size :

$$r_H(z_{eq}) = \int_0^{t_{eq}} \frac{c\,dt}{a} = c\int_0^{a_{eq}} \frac{da}{H(a)a^2}$$

Radiation dominated era: $H^2(a) = H_0^2\left[\Omega_M a^{-3} + \Omega_{rad} a^{-4}\right]$

$$r_H(a_{eq}) = \frac{c}{H_0} 2(\sqrt{2}-1)\frac{1}{\sqrt{\Omega_M z_{eq}}}$$

Known from T_{CMB}

$$= 2(\sqrt{2}-1)\frac{c}{70km/s/Mpc}\frac{\sqrt{\Omega_{rad} h_{70}^2}}{\Omega_M h_{70}^2}$$

"The matter density is imprinted on the sky"

Interlude: measuring the matter correlation function from galaxies

"Correlation function" has nothing to do with "correlation coefficient".

Do it yourself (!):

1) Find a (large) telescope and measure the positions (2 angles) and redshifts of, say 50,000 galaxies (!)

2) Compute the distances between all galaxy pairs and plot a histogram of it (& account for acceptance)

Warning: statistically correlated error bars