The Standard Model and beyond (2)
QCD

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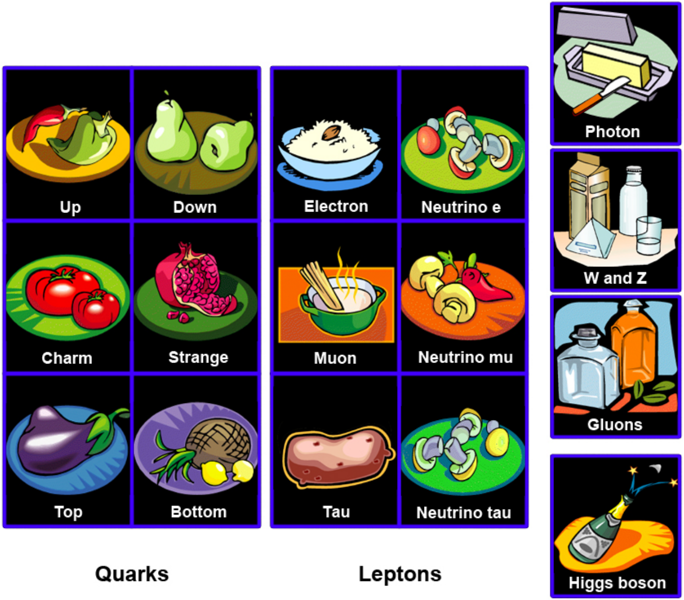
LPT Orsay



Fermions **Bosons**

First lecture

Quantum Field Theory

- Combining quantum mechanics and special relativity
- Fields able to create and annilate particles at points of space time
- Relativistic Lagrangian for spins 0 (Klein Gordon) and 1/2 (Dirac)
- In QFT to compute transition amplitudes from a state to another...
- ...involving intermediate states with different number of particles, not necessarily allowed in a classical theory

Quantum Electrodynamics

- Free classical Lag: Maxwell (em field A_μ) and Dirac (fermion ψ)
- Gauge principle: global invariance of Dirac theory (phase redef of ψ) into local (phase rotation depending on space-time position)
- Covariant derivative $D_\mu\psi$ involves a new spin 1 field, identified with A_μ, coupling electrons (associated with ψ) and photons (A_μ)
- Tested to a high accuracy: $(g-2)_\mu$, variation of α with energy

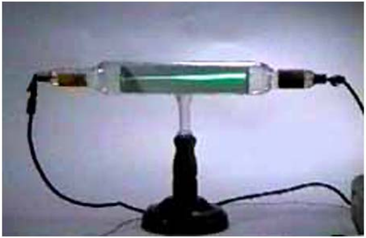
A more complicated kind of fermions
the quarks

From leptons to quarks

Started with leptons, now we move to quarks

- constitute hadrons : baryons (qqq) and mesons ($q\bar{q}$)
- have flavours and colours

Flavours



- 1930s - Protons, neutrons, electrons

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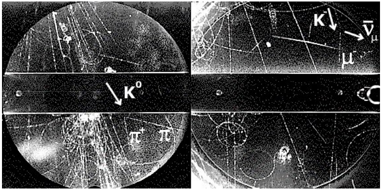

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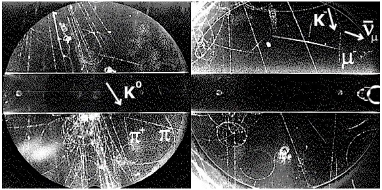

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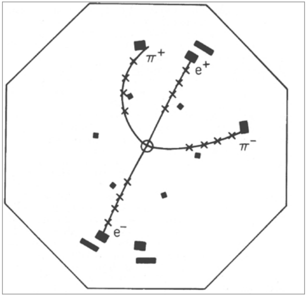

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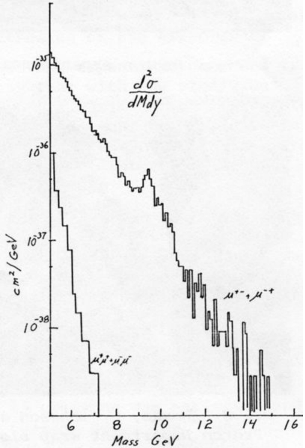

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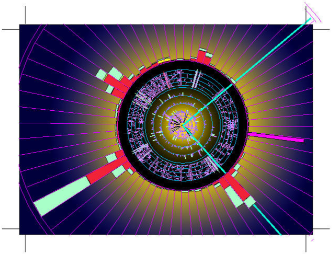

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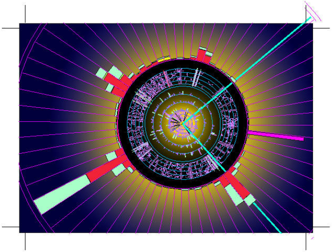

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6 flavours arranged in 3 generations, more and more massive

- 1 up-type quark ($Q = 2/3$) u, c, t
- 1 down-type quark ($Q = -1/3$) d, s, b

Colours

- Quark model : proton *uud*, neutron *udd*...
- Among states discovered in 50's
  $\Delta^{++}(J=3/2, J_3=3/2) = u^\uparrow u^\uparrow u^\uparrow$
- But Δ is a fermion, with antisymmetric wave function (Pauli)

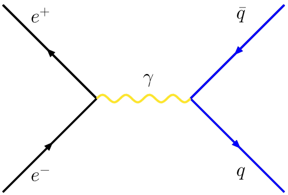
$\Longrightarrow$additional d.o.f. : colour (green, blue, red)

$$\Delta^{++}(J=3/2, J_3=3/2) = \epsilon^{\alpha\beta\gamma} u^\uparrow_\alpha u^\uparrow_\beta u^\uparrow_\gamma$$

More generally, if i, j, k flavour and α, β, γ colour, hadrons combine quarks in colourless combination

- Baryons consist of $\epsilon^{abc} q^i_\alpha q^j_\beta q^k_\gamma$
- Mesons consist of $\delta^{\alpha\beta} q^i_\alpha \bar{q}^j_\beta$
- Exotics ?

How many colours ?

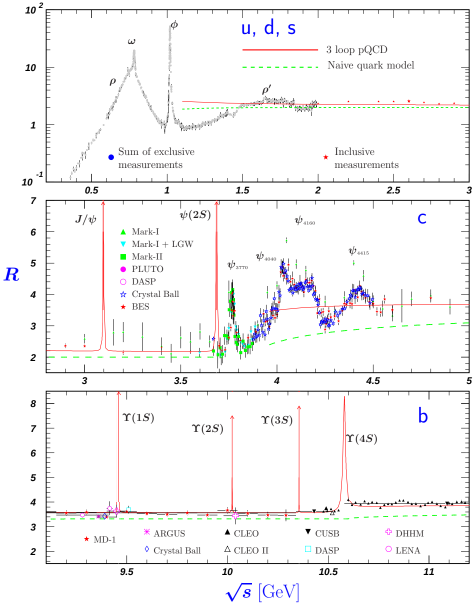


$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq N_c \sum_q Q_q^2$$

$$= \begin{cases} 2/3 \cdot N_c & (u,d,s) \\ 10/9 \cdot N_c & (u,d,s,c) \\ 11/9 \cdot N_c & (u,d,s,c,b) \end{cases}$$

vary when a $q\bar{q}$ threshold production is crossed

3 colours



Resonances after each $q\bar{q}$ threshold, then asymptotic value with $N_c = 3$

What can we do with quarks having flavours and colours ?

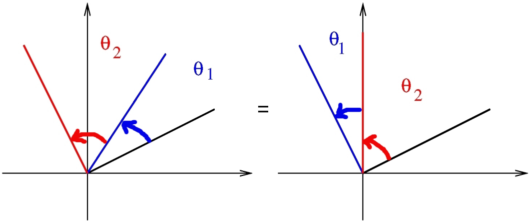
A short detour through symmetries

Symmetries

- In QED, symmetry under phase redefinition

$$\psi \to e^{i\alpha Q}\psi$$

- $U(1)$ equivalent to $O(2)$ symmetry, rotations in 2 dimensions



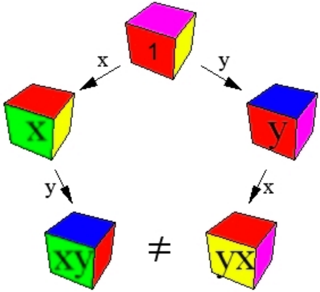
abelian (i.e.m commuting) group:

$$R(\theta_1)R(\theta_2) = R(\theta_2)R(\theta_1) = R(\theta_1 + \theta_2)$$

Not always the case !

Nonabelian symmetries

Rotations in larger spaces are nonabelian,
for instance $O(3)$: rotations and reflexions in 3 dimensions



- A group: $R_1 R_2$ still a rotation, belongs to $O(3)$
- But not abelian: $R_1 R_2 \neq R_2 R_1$
- Structure of the group specified by $[R_1, R_2] = R_1 R_2 - R_2 R_1$

Group transformation

- Representation of the group : "how the object transforms"

  For instance, under a $SO(3)$ (three-dimensional) rotation
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$$J^a = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

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  - spinor ψ : $\psi^\alpha \to [S_{1/2}(R)]^{\alpha\beta} \psi^\beta \equiv [\exp[-i\theta_a \sigma^a/2]]^{\alpha\beta} \psi^b$

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- Lie Algebra : "how the group is characterised" (indep of repres.)

  $U = \exp(-i\theta_a T^a)$ with T^a traceless hermitian generators
  where $[T^a, T^b] = i f^{abc} T^c$ $\qquad$ f^{abc} group structure csts

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  Rotations : $\qquad [J^a, J^b] = i\epsilon^{abc} J^c \qquad [\sigma^a/2, \sigma^b/2] = i\epsilon^{abc}\sigma^c/2$

  $\Longrightarrow$Infinitesimal version of the "table of multiplication" of the group

$SU(2)$ and $SU(3)$ groups

$U = \exp(-i\theta_a T^a) \in SU(N)$ parametrised by θ_a

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  Fundamental represent. $T^a = \frac{1}{2}\sigma^a$ from Pauli matr ($f^{abc} = \epsilon^{abc}$)

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- $SU(3)$: $a = 1 \ldots 8$ matrices 3×3
  Fundamental represent. $T^a = \frac{1}{2}\lambda^a$ from Gell-Mann matrices

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \ldots$$

Flavour symmetry

For light flavours (u, d, s)

$$\mathcal{L}_D = \bar{\Psi}(i\gamma^\mu \partial_\mu - M)\Psi \qquad \Psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

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In the limit where $m_u = m_d$ $[m_u = m_d = m_s]$,

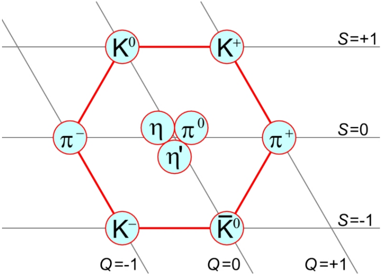
$\mathcal{L}_D$ isospin symmetric $SU_F(2)$ [flavour symmetric $SU_F(3)$]

$$\mathcal{L}_D \to \mathcal{L}_D \text{ if } \Psi \to U\Psi \text{ and } \bar\Psi \to \bar\Psi U^\dagger$$ (global redefinition of u, d, s)

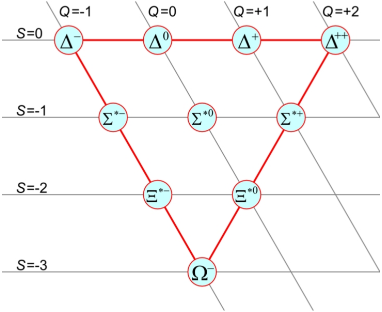
with U an $N_f \times N_f$ special unitary matrix: $UU^\dagger = U^\dagger U = 1, \quad \det U = 1$

The eightfold way

SU(3) (global) flavour symmetry: u, d, s equivalent for strong forces
$\Longrightarrow$almost degenerate spectrum (hadrons bound states of quarks)
organised in multiplets given by $SU(3)$ representations



Nonet of mesons



Decuplet of baryons

octets and decuplets with approximately identical masses
(also used to relate processes for different members of multiplets)

Colour

Colour symmetry

- Free coloured quarks $q = \begin{pmatrix} q \\ q \\ q \end{pmatrix}$ $\qquad \mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q$

  with a global colour symmetry $q(x) \to Uq(x) = \exp[i\alpha_a \lambda^a/2]q(x)$
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  $G^\mu_{\alpha\beta} = G^\mu_a(\lambda^a)_{\alpha\beta}/2$ collects eight gluons !

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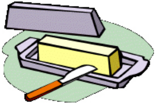

QED	QCD
One phase	Three colours
$U(1)$	$SU(3)$
Abelian symmetry	Nonabelian symmetry
1 parameter	8 parameters

QCD Lagrangian

Invariance under local colour rotations yields QCD Lagrangian

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- A term for the quarks : free + interaction

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- but also a kinetic term for the gluons

$$\mathcal{L}_F = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} = -\frac{1}{2}Tr[G^{\mu\nu}G_{\mu\nu}]$$

where $G^{\mu\nu}$ is the analogue of electromagnetic $F^{\mu\nu}$

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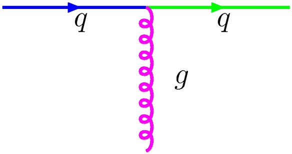
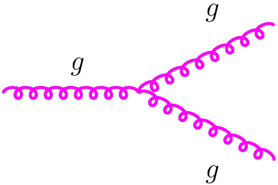
- No mass term (not gauge invariant), hence gluons are massless
- Interactions: q-q-g from $\mathcal{L}_D$, 3 gluons and 4 gluons from $\mathcal{L}_F$ [new !]

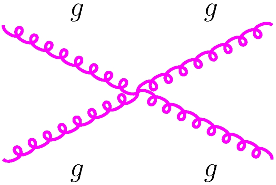
QCD interactions



$g_s \gamma^\mu \lambda_{\alpha\beta}/2$



$g_s f^{abc}$



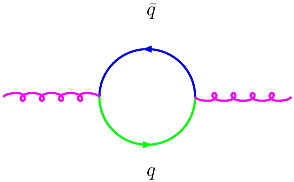
$g_s^2 f_{abc} f_{ade}$

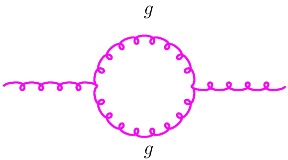
Differences from electromagnetism

- Gluons themselves sensitive to strong interaction
- Universal coupling g_s (no "colour-electric charge")

Consequences of QCD for strong interaction

And vacuum polarisation, e.g. gluon exchange between 2 quarks ?





Asymptotic freedom

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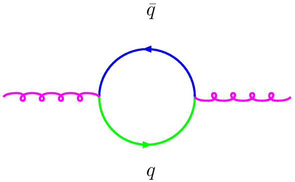

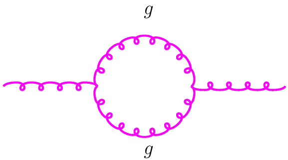

Pairs of virtual quarks AND gluons from the vacuum

- modification of $\alpha_s = g_s^2/(4\pi)$ with the distance/energy

$$\frac{dg_s(q)}{d\log(q)} = \beta(g) = -\frac{g^3}{4\pi^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right] + \ldots$$

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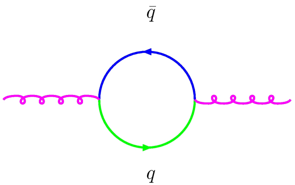
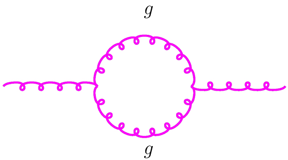
$$\frac{dg_s(q)}{d\log(q)} = \beta(g) = -\frac{g^3}{4\pi^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right] + \ldots$$

- N_f from quarks : α_s increases at small distances (large q)

Asymptotic freedom

And vacuum polarisation, e.g. gluon exchange between 2 quarks ?





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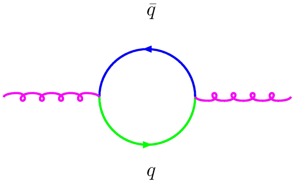

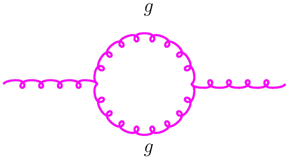

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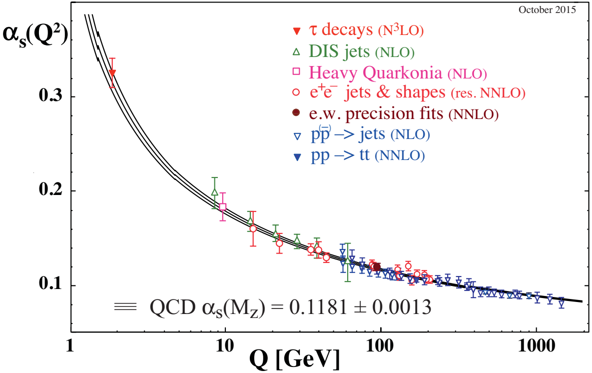
- N_f from quarks : α_s increases at small distances (large q)
- N_c from gluons : α_s decreases at small distances
- in our world ($N_c = 3$, $N_f = 6$), the gluons win and $\beta < 0$!

α_s decrease at small distances

α_s at various scales

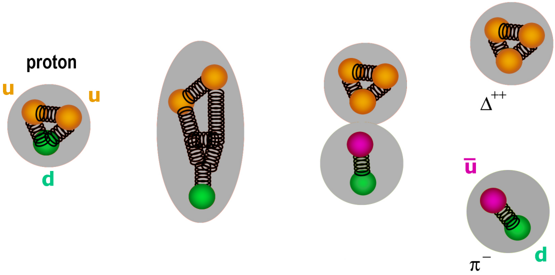


$\Longrightarrow$ asymptotic freedom:
at large energies, interactions (prop to g_s) small perturbations

Consistency over a very large range of energies
(from m_τ up to LHC pp collisions)

Confinement

At distances of order 1 fm, α_s becomes of $O(1)$, $V_{qq}(r) \sim r$ (not $1/r$!)



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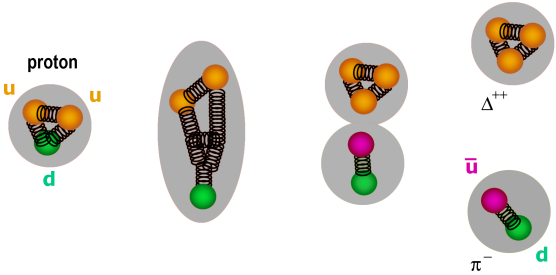

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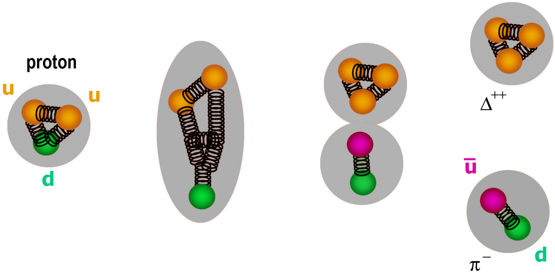

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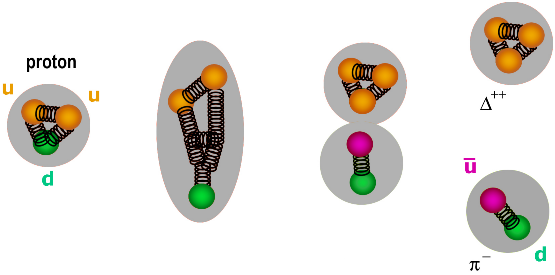

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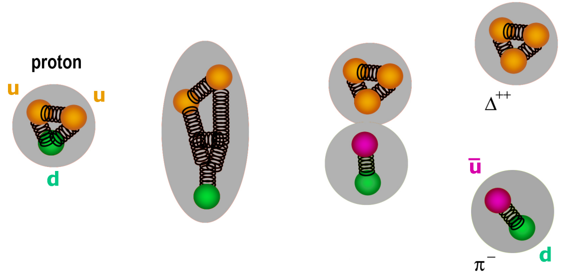

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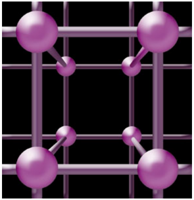
- No perturbation theory possible for soft physics (below 1 GeV)
- Often processes mixture of strong and electroweak
  $\Longrightarrow$quark decays weakly into another quark inside a hadron
- Hard to connect theory (quarks) and experiment (hadrons)
  - solve numerically the equations (lattice gauge theory)
  - build a theory of more limited scope (effective field theory)

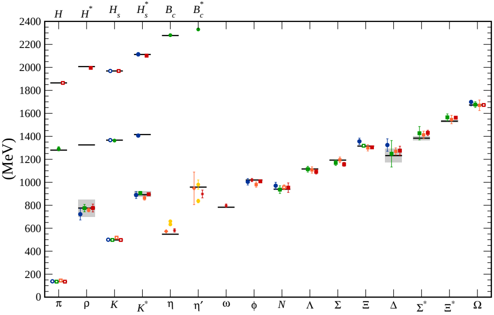
Lattice gauge theories



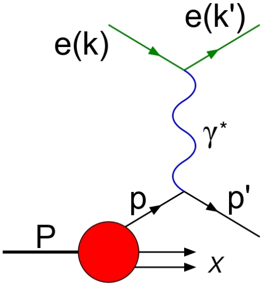
Compute propagation and decay of a particle

- Discretise space and time (lattice spacing)
- Finite 4D box (finite-volume effects) with Euclidean metric
- Sum over all possible configurations (Monte Carlo methods)

Recent progress in understanding effect of (virtual) sea quarks, finite volume, lattice spacing and renormalisation. . .



Deep inelastic scattering: parton model



$$e^-(k)p(P) \to e^-(k') + X$$

2 kinematic variables

$$x = -\frac{q^2}{2P\cdot q}, y = \frac{P\cdot q}{P\cdot k} \qquad (q = k - k')$$

In parton model
energetic proton made of
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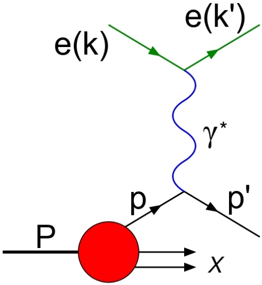

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$$\frac{d^2\sigma}{dxdy} = \sum_f [x f_f(x) Q_f^2] \times \frac{4\pi\alpha^2(P\cdot k)}{q^4}[1 + (1-y)^2]$$

$f_f(x)$: parton distribution function, probability of finding a constituent f with a longitudinal fraction x of momentum

$\Longrightarrow$ Parton model: scaling of the cross section with x

Deep inelastic scattering : QCD

$$e^-(k)p(P) \to e^-(k') + X$$

$$q = k - k' \qquad x = -\frac{q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot k}$$

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QCD provides corrections to the parton scaling

$$\frac{d^2\sigma}{dxdy} = \sum_f [x f_f(x, q) Q_f^2] \times \frac{4\pi\alpha^2 (P \cdot k)}{q^4} [1 + (1-y)^2] + O(\alpha_s)$$

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Deep inelastic scattering : QCD

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$f_f(x,q)$: probability of finding a constituent f
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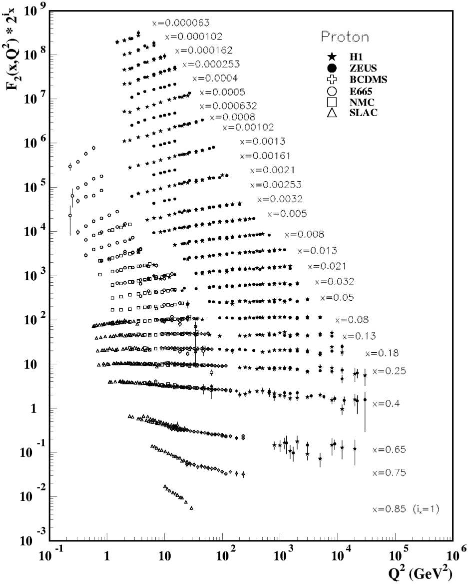
Two types of QCD correction

- $O(\alpha_s)$ and higher-order corrections to vertex
- variation of $f_f(x,q)$ with q



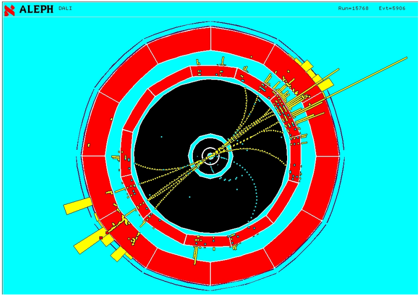
Measurements of

$$F_2 = \sum_f x Q_f^2 f_f(x, q)$$

Variations with q
in agreement with QCD

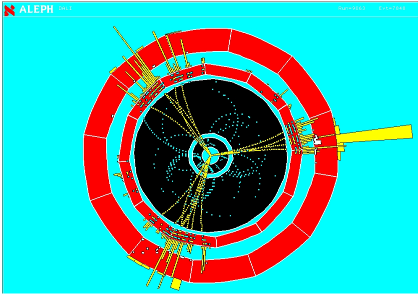
In collisions, quarks/gluons emit further gluons/quarks and lose energy, until they become soft (around 1 GeV) and bind into hadrons

$e^+e^- \to q\bar{q}$



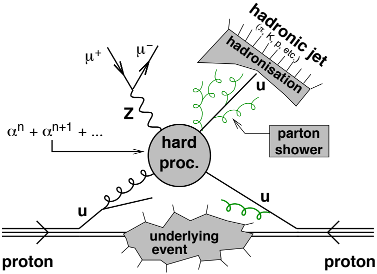
Two jets

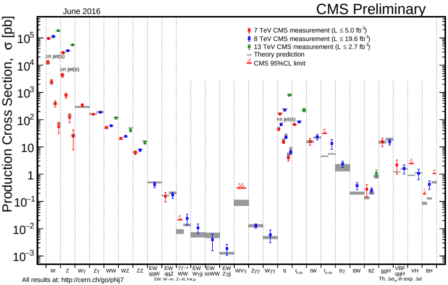
$e^+e^- \to q\bar{q}g$



Three jets

$\Longrightarrow$Global observables, dependent on high energies (infrared safe), well described by perturbative QCD : total σ, thrust, sphericity





- Separation of scales between hard (perturbative) and soft (hadronic) dynamics
- Probe QCD and approximate models for Monte Carlo simulations
- Constraining α_s and/or parton distribution functions
- Good agreement with NLO QCD over 11 orders of magnitude
- Next steps: NNLO (already for $t\bar{t}$ production), processes with H

End of part II