The Standard Model and beyond (4)
Electroweak symmetry breaking and the Higgs boson

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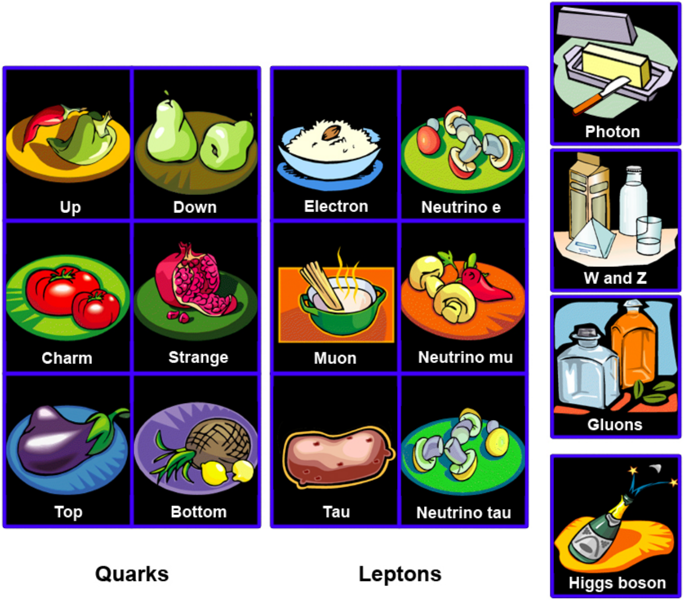


Quarks

Leptons

Fermions

Bosons

Third lecture

Charge and Parity conjugation

- Discrete symmetries
- Obeyed by electromagnetic and strong interactions, but not weak
- Connection with left and right-handed chiralities of fermions ?

Weak interaction

- Charged-current transitions involving only left-handed fermions
- Neutral-current transitions involving neutrinos, no flavour change
- Distinguishing left-handed doublets and right-handed singlets
- Symmetry group $SU(2)_L \times U(1)_Y$ promoted to local symmetry
- 3 W_μ bosons and 1 B_μ leading to observed $W^\pm$ (charge current for left-handed), A_μ and Z_μ (neutral current for left and right-handed)
- with given couplings related to QED couplings
- symmetry too powerful: no mass terms for bosons and fermions

Symmetry breaking

The trouble with the theory

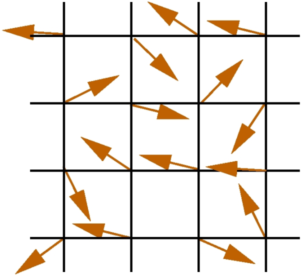
$SU(2)_L \times U(1)_Y$ a wonderful theory ?

- Description of weak and electromagnetic gauge bosons
- Universality of the couplings
- Chiral theory (left- and right-hd fermions are different)
- …but fermion and boson masses forbidden by gauge symmetry !

We should keep enough symmetry for gauge interactions,
but break enough to allow for masses in the spectrum

Ordered and unordered phase

Take a ferromagnet above the Curie temperature, without external field

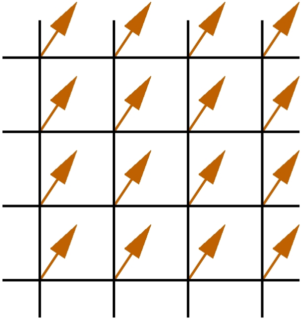


Spin-spin interaction $\vec{S}_i \cdot \vec{S}_j$
invariant under rotations $O(3)$

No order and no privileged direction

Spontaneous breakdown of symmetry

Under Curie temperature : Spontaneous magnetisation $\langle \sum_j \vec{S}_j \rangle \neq \vec{0}$



Privileged direction for states, although no dir. privileged in interaction

Spontaneous breakdown of symmetry

symmetries of interactions not explicit in states

[interactions] $O(3) \to O(2)$ [states]

An example closer to particle physics

Classically, take a spin-0 complex field ϕ $\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - V(\phi)$

with a scalar potential V containing mass term + interactions

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

V is invariant under reparametrisation $\phi(x) \to e^{i\alpha}\phi(x)$

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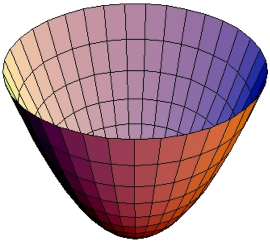
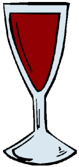


For $\lambda > 0$ and $\mu^2 > 0$

Trivial minimum $\phi_0 = 0$
invariant under phase
reparametrisation

Spectrum : a charged
particle (and its antiparticle)
with the mass $m_\phi = \mu$



Two directions

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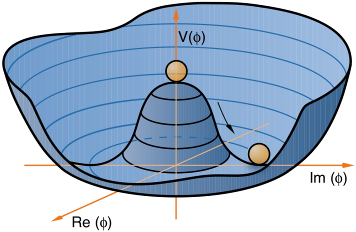
ring of nontrivial minima

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} > 0$$



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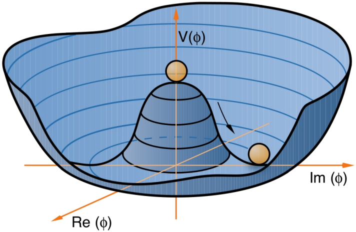

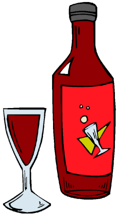

Spontaneous breaking of rotation symmetry
by the fundamental state

To determine the spectrum of the theory

- Determine the (nontrivial) vacuum
- Expand the theory around it

Spectrum of the quantum theory

- Minimum given by vacuum expectation value of ϕ: $\langle 0|\phi|0\rangle = v$

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- Express the Lagrangian in terms of real $\phi_{1,2}$

$$\begin{aligned}\mathcal{L} &= \partial^\mu\phi^\dagger\partial_\mu\phi - V(\phi)\\ &= \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\left(1+\frac{\phi_1}{v}\right)^2\partial_\mu\phi_2\partial^\mu\phi_2\\ &\qquad - \left[V(\phi_0) + \frac{1}{2}m_{\phi_1}^2\phi_1^2 + \lambda v\phi_1^3 + \frac{1}{4}\lambda\phi_1^4\right]\end{aligned}$$

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This was for a global rotation symmetry
what about the local electroweak symmetry ?

Electroweak symmetry breaking

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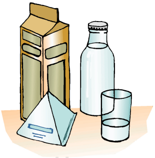
- Write down its Lagrangian

$$\mathcal{L}_H = (D_\mu \phi)^\dagger (D_\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$
$$D^\mu \phi = [\partial^\mu - igW^\mu - ig' y_\phi B^\mu]\phi$$



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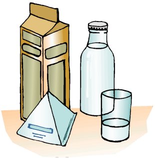

- $\lambda > 0$ and $\mu^2 < 0$ yields degenerate vacuum states

$$|\langle 0|\phi^{(0)}|0\rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} \qquad |\langle 0|\phi^{(+)}|0\rangle| = 0$$

with direction chosen not to break gauge invariance for electromag

How to proceed

- Take the previous theory for electroweak unification
- Add the symmetry breaking part involving ϕ
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- Check the phenomenological consequences

Masses of the gauge bosons

Kinematic terms of ϕ contains covariant derivatives coupling to W, Z

$$
\begin{aligned}
\mathcal{L}_H &\ni (D_\mu\phi)^\dagger(D^\mu\phi) \\
&= \frac{1}{2}\partial_\mu H\partial^\mu H + \frac{g^2}{4}(v+H)^2[W_\mu^\dagger W^\mu + \frac{1}{2}\cos^2\theta_W Z_\mu Z^\mu] \\
&\to \frac{g^2 v^2}{4}[W_\mu^\dagger W^\mu + \frac{1}{2}\cos^2\theta_W Z_\mu Z^\mu]
\end{aligned}
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reexpressed in terms of physical states A, W, Z

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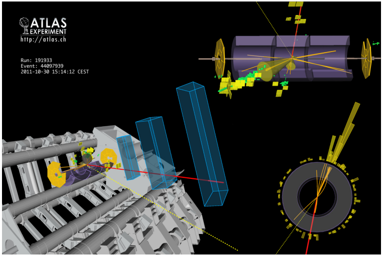
reexpressed in terms of physical states A, W, Z

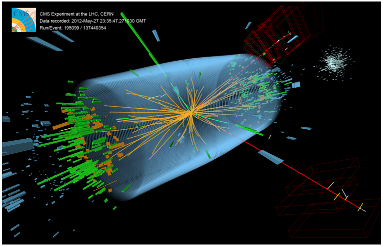
- Em gauge invariance: no $A_\mu A^\mu$ term, photon massless,
- Massive weak gauge bosons: $M_Z\cos\theta_W = M_W = \frac{1}{2}vg$
- Symmetry breaking: $SU(2)_L \times U(1)_Y \to U(1)_{em}$

Higgs mechanism

Massless $W^\pm, Z$		Massive $W^\pm, Z$
(3 $\times$ 2 polarisations)	$\Longrightarrow$	(3 $\times$ 3 polarisations)
4 scalar d.o.f. $\vec{\theta}, H$		1 scalar d.o.f. H



$pp \to W^+W^- + X \to \mu^-\bar{\nu}_\mu q\bar{q}'$



$pp \to Z^0Z^0 + X \to e^+e^-\mu^+\mu^- + X$

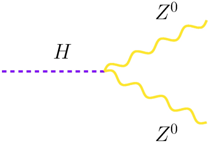
with one massive Higgs H remaining as an observable particle !

Higgs mech. (W, Z eating Goldstone) $\neq$ Higgs boson (dinner leftovers)

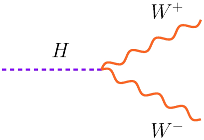
Coupling of H to the gauge bosons

Scalar Lagrangian contains Higgs interactions with the gauge bosons

$$\mathcal{L}_H \ni \left[m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu\right]\left[1 + \frac{2}{v} H + \frac{H^2}{v^2}\right]$$



$2m_Z^2/v$



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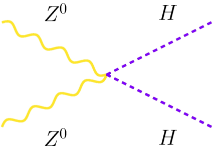

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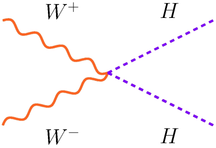

m_W^2/v

Coupling of H to the fermions and their masses

Gauge symmetry allows Yukawa coupling of scalar ϕ with 2 fermions:
1 left-handed doublet and 1 right-handed singlet

$$\mathcal{L}_Y = (\bar{q}_u \ \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right]$$

$$+ (\bar{\nu}_\ell \ \bar{\ell})_L c^{(\ell)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \ell_R + h.c.$$

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yielding, once reexpressed around vacuum

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) [m_{q_d} \bar{q}_d q_d + m_{q_u} \bar{q}_u q_u + m_\ell \bar{\ell}\ell]$$

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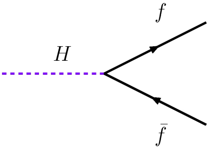

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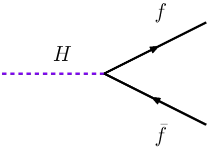

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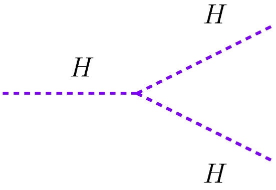
Higgs field provides masses to gauge bosons and fermions

Self couplings of the Higgs boson

In the scalar Lagrangian, depending on 2 free parameters μ and λ one finds a part for the Higgs boson itself

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2} m_H^2 H^2 - \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4$$

- $v = \sqrt{-\mu^2/\lambda}$ fixed from ew symmetry breaking ($M_W, M_Z \ldots$)
- mass $m_H = \sqrt{2\lambda} v$ free parameter to be fixed experimentally



3-Higgs coupling
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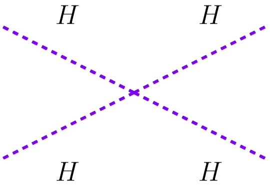

4-Higgs coupling
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The Higgs boson

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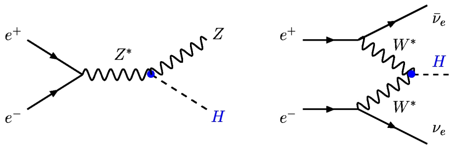

Leptonic machines

- $e^+e^- \to Z^* \to Z + H$
- $e^+e^- \to \bar{\nu}\nu W^* W^* \to \bar{\nu}\nu H$

Finding the Higgs boson

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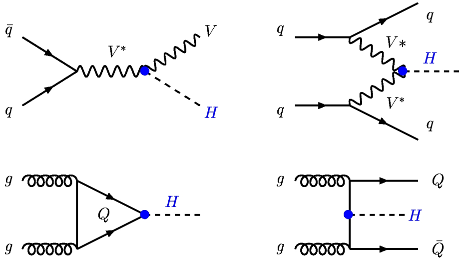

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Hadronic machines

- $q\bar{q} \to V + H$
- $qq \to V^* V^* \to qq + H$
- $gg \to H$
- $gg, q\bar{q} \to Q\bar{Q} + H$

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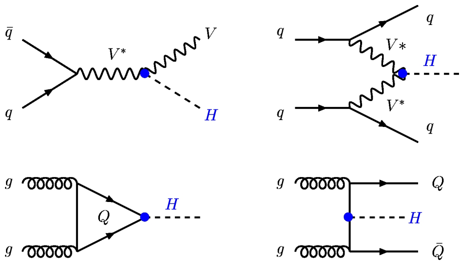

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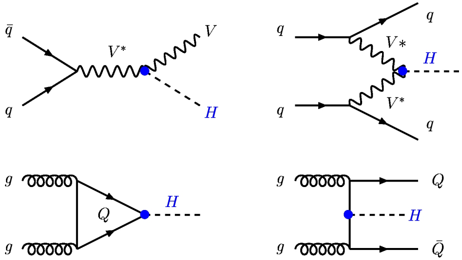
- $gg \to H$
- $gg, q\bar{q} \to Q\bar{Q} + H$

with a preference for decays into heavy particles

Finding the Higgs boson

From the above interactions, Higgs boson produced in different ways



Leptonic machines

- $e^+e^- \to Z^* \to Z + H$
- $e^+e^- \to \bar{\nu}\nu W^* W^* \to \bar{\nu}\nu H$

Hadronic machines

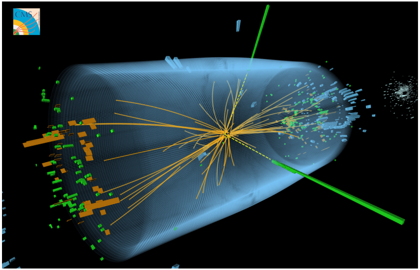
- $q\bar{q} \to V + H$
- $qq \to V^* V^* \to qq + H$
- $gg \to H$
- $gg, q\bar{q} \to Q\bar{Q} + H$

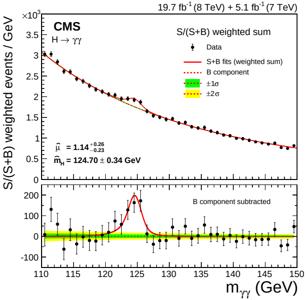
with a preference for decays into heavy particles

One of the main objectives of ATLAS and CMS experiments at LHC and a success announced in 2012 !

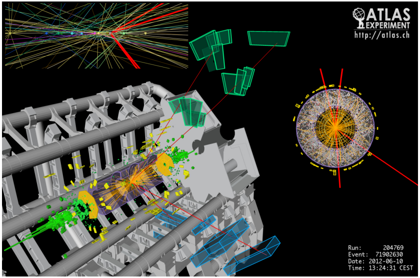
Seeing the Higgs boson

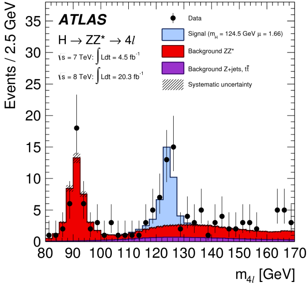
$H \to \gamma\gamma$





$H \to ZZ \to 4\ell$



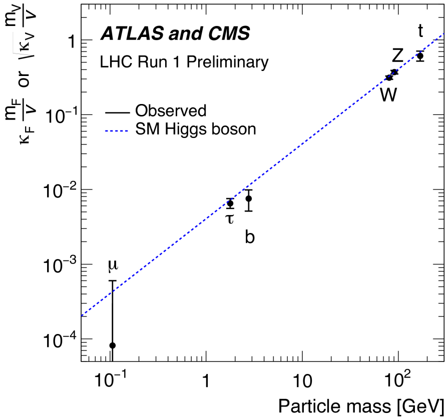


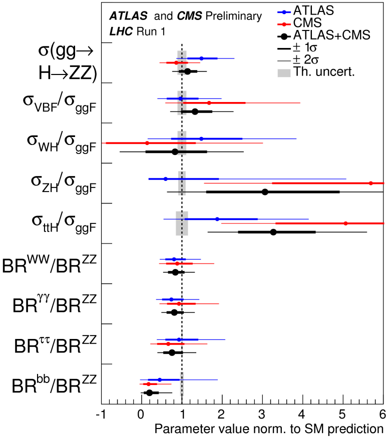
Identifying the Higgs boson

	SM Br (%)	Signif ATLAS (σ)	Signif CMS (σ)
$H \to b\bar{b}$	58.4 ± 1.9	1.4	2.1
$H \to WW$	21.4 ± 0.9	6.5	4.7
$H \to \tau\tau$	6.27 ± 0.36	4.5	3.8
$H \to ZZ$	2.62 ± 0.11	8.1	6.5
$H \to \gamma\gamma$	0.227 ± 0.011	5.2	4.6
$H \to \mu\mu$	0.018 ± 0.001	-	-

- From $H \to \gamma\gamma$ and $H \to ZZ$ (run 1): $m_H = 125.09 \pm 0.21 \pm 0.11$ GeV
- From $H \to ZZ \to 4\ell$: $J^P = 0^+$ (other hyp excluded at 99% CL)

Cross checking Higgs properties

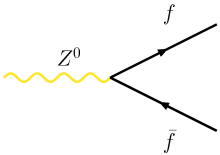


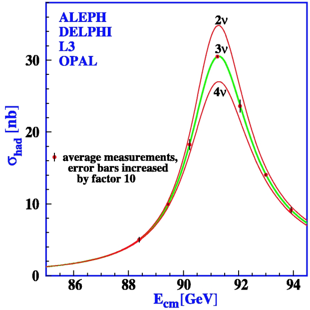


- Higgs coupling prop to mass of spin-1 and spin-1/2 masses at tree
- Production (σ) or decay (Br) with respect to SM

A few more tests of the Standard Model

Z coupling to neutrinos





$$\Gamma(Z \to f\bar{f}) \propto |v_f|^2 + |a_f|^2$$

$$\frac{\Gamma(Z \to \text{invisible})}{\Gamma(Z \to \ell^+\ell^-)} = N_\nu \frac{\Gamma(Z \to \nu_\ell \bar{\nu}_\ell)}{\Gamma(Z \to \ell^+\ell^-)} = N_\nu \frac{2}{1 + (1 - 4\sin^2\theta_W)^2} = 1.96 N_\nu$$

LEP measurements: Only 3 light neutrinos !

Z related observables

Consider the cross section for $e^+e^- \to \gamma, Z \to f\bar{f}$
with an angle θ between in and out states in center of mass

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_{em}^2}{8s} N_f [A(1+\cos^2\theta) + B\cos\theta - h_f[C(1+\cos^2\theta) + D\cos\theta]]$$

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- N_f number of fermion species: $N_\ell = 1, N_q = N_c(1 + \alpha_s(M_Z^2) + \ldots)$
- h_f helicity of the fermion

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- A, B, C, D predicted functions of Z couplings : a_e, v_e, a_f, v_f

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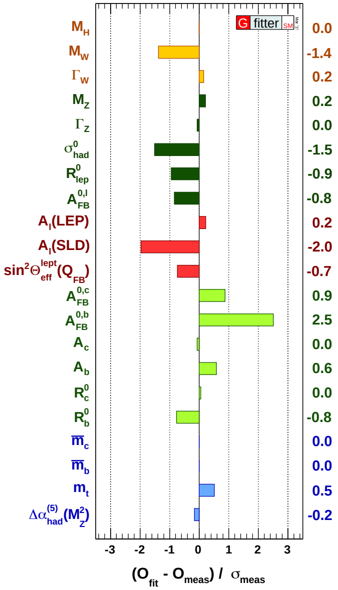
$$\sigma = \frac{4\pi\alpha_{em}^2}{3s} N_f A \qquad A_{FB}^f = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\frac{B}{A}$$

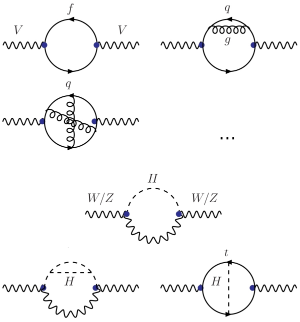
$$A_{LR}^f = \frac{\sigma^{h_f=1} - \sigma^{h_f=-1}}{\sigma^{h_f=1} + \sigma^{h_f=-1}} = -\frac{C}{A}$$

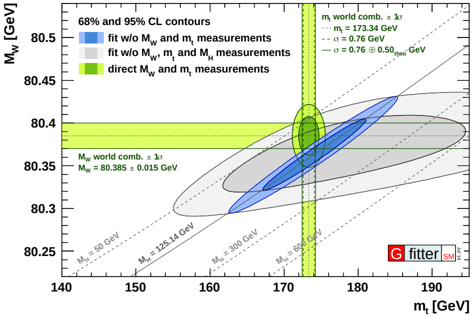
- At the Z peak, $A_{FB}^f = \frac{3}{4} A_{LR}^e A_{LR}^f$ (measures polarisation of quarks)

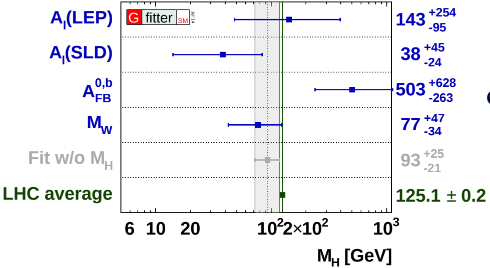
Electroweak precision measurements



- Fitting the previous observables and others, depending on M_H and m_t
- Good overall agreement







- m_t, m_H and m_W in good agreement from electroweak observables
- m_H determined indirectly from electroweak observables in good agreement with direct determination
- Important constraint for any theory beyond the Standard Model

Fermion mass matrices

- Yukawa interactions, but 3 generations

Fermion mass matrices

- Yukawa interactions, but 3 generations yield 3×3 matrices

$$\sum_{i,j=1,2,3} (\bar{q}'_d)^i_L (M_d)_{ij} (q'_d)^j_R + (\bar{q}'_u)^i_L (M_u)_{ij} (q'_u)^j_R + (\bar{\ell}')^i_L (M_\ell)_{ij} (\ell')^j_R$$

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- Mass states ? Diagonalise $M_f = V_f^\dagger m_f U_f$
  where U and V unitary, and m diagonal

$$[(\bar{q}_d)_L m_d (q_d)_R + (\bar{q}_u)_L m_u (q_u)_R + \bar{\ell}_L m_\ell \ell_R + h.c.]$$

with mass eigenstates q from interaction eigenst. q' via unitary rot

$$\begin{array}{ccc} (q_d)_L = V_d (q'_d)_L & (q_u)_L = V_u (q'_u)_L & \ell_L = V_\ell \ell'_L \\ (q_d)_R = U_d (q'_d)_R & (q_u)_R = U_u (q'_u)_R & \ell_R = U_\ell \ell'_R \end{array}$$

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- Interactions defined in terms of q'
  introducing U, V in interactions when expressed in terms of q

Charged & neutral currents

- Flavour-conserving neutral: $\bar{f}_L \Gamma f_L = \bar{f}'_L \Gamma f'_L$, $\bar{f}_R \Gamma f_R = \bar{f}'_R \Gamma f'_R$

$$\mathcal{L}_{NC} = \frac{e}{\sin(2\theta_W)} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f$$

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- Flavour-changing charged: $\bar{u}'_L \Gamma d'_L = \bar{u}_L V_u \Gamma V_d^\dagger d_L = \bar{u}_L V^{CKM} \Gamma d_L$

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij}^{CKM} d_j + \sum_i \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \ell_i \right] + h.c.$$

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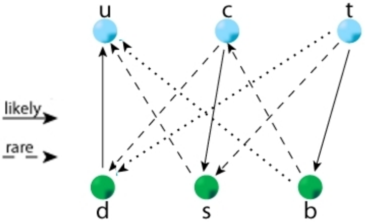
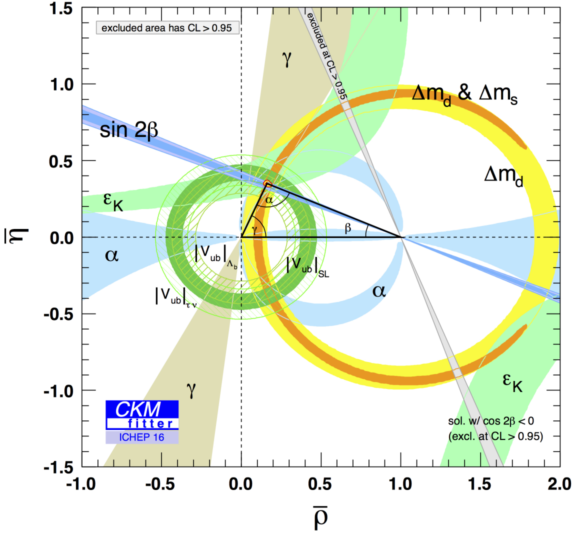


- For 3 generations, Cabibbo-Kobayahi-Maskawa matrix V contains one imaginary term, (only) source of CP violation in SM
- If no ν_R, $m_\nu = 0$, ℓ rotation absorbed in ν, no lepton mixing matrix

The CKM matrix



- V^{CKM} depends on 4 parameters $A, \lambda, \bar\rho, \bar\eta$
- Each band is a constraint from one (or several) weak process involving quarks
- Agree, lead to accurate $\bar\rho, \bar\eta$
- $\bar\eta \neq 0$ indicates CP-violation

Important constraint for any theory beyond the Standard Model

End of part IV